

MATHEMATICS 360-255-LW

Quantitative Methods II

Martin Huard

Fall 2009

XX – Inferences about the Population Variance

1. The manufacturer of a certain brand of light bulbs claims that the variance of the lives of these bulbs is 4000 square hours. A consumer agency took a random sample of 25 such bulbs and tested them. The variance of the lives of these bulbs was found to be 4990 square hours. Assume that the lives of all such bulbs are (approximately) normally distributed.
 - a) Make the 99% confidence interval for the variance and standard deviation of the lives of all such bulbs.
 - b) Test at the 1% significance level if the variance of such bulbs is different from 4000 square hours. Try with both approaches, the classical and the p -value.

2. The following are the prices of the same brand of camcorder found at eight stores in Montreal.

\$749 815 789 799 732 825 799 769

- a) Make the 95% confidence interval for the population standard deviation.
 - b) Test at the 5% significance level if the variance of the population is different from 500 square dollars. Try with both approaches, the classical and the p -value.
3. In 1941, the population variance for the age (in years) of a rural Quebec woman at the time of her first marriage was approximately 5.1. A recent study of age of first marriage for a random sample of 41 women in rural Quebec gave a sample variance of 3.3.
 - a) Make the 90% confidence interval for the variance and standard deviation for the recent age of first marriage of women in rural Quebec.
 - b) Test at the 10% significance level the claim that the current variance is less than 5.1. Try with both approaches, the classical and the p -value.
 4. A machine that puts corn flakes in boxes is adjusted to put an average of 400 grams in each box with a standard deviation of 6 grams. A random sample of 12 boxes gave a sample standard deviation of 9 grams.
 - a) Make the 95% confidence interval for the standard deviation for the volume in corn flakes boxes.
 - b) Do these data support the claim that the variance has increased and the machine needs to be brought back into adjustment? Try with both approaches, the classical and the p -value.

ANSWERS

1. a) variance: 2629 squared hour to 12114 squared hours
 standard deviation: 51 hours to 110 hours
- b) $H_o : \sigma^2 = 4000$ critical values: $\chi^2_{(24,0.995)} = 9.89$ Fail to reject H_0
 $H_A : \sigma^2 \neq 4000$ $\chi^2_{(24,0.005)} = 45.56$
 test statistic: $\chi^2 = 29.94$
 $0.20 < p - \text{value} < 0.50$
2. a) \$21.30 to \$65.60
- b) $H_o : \sigma^2 = 500$ critical values: $\chi^2_{(7,0.975)} = 1.69$ Fail to reject H_0
 $H_A : \sigma^2 \neq 500$ $\chi^2_{(7,0.025)} = 16.01$
 test statistic: $\chi^2 = 14.54$
 $0.05 < p - \text{value} < 0.10$
3. a) variance: 2.37 squared years to 4.98 squared years
 standard deviation: 1.54 years to 2.23 years
- b) $H_o : \sigma^2 = 5.1$ critical value: $\chi^2_{(40,0.90)} = 29.05$ Reject H_0
 $H_A : \sigma^2 < 5.1$ test statistic: $\chi^2 = 25.88$
 $0.025 < p - \text{value} < 0.05$
4. a) 6.38 grams to 15.28 grams
- b) $H_o : \sigma^2 = 36$ critical value: $\chi^2_{(11,0.05)} = 19.68$ Reject H_0
 $H_A : \sigma^2 > 36$ test statistic: $\chi^2 = 24.75$
 $0.050 < p - \text{value} < 0.010$