

## MATHEMATICS 360-255-LW

Quantitative Methods II

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# XVIII – Chi-Square Test of Independence

1. A random sample of 1000 was asked whether they voted in the last general election. Here are the results, broken down by age group.

	18-24 years	25-39 years	40-59 years	60 years or older
Voted	46	159	178	159
Did not vote	90	179	118	71

At the 1% level of significance, test the claim that voting status and age are independent. Try with both approaches, the classical and the  $p$ -value.

2. Here are the results of a random sample of 535 adults.

Education Level	Smoker	Nonsmoker
Less than high school	47	96
High school graduate	56	117
College graduate	27	81
University graduate	17	94

At the 5% level of significance, test the claim that smoking status is independent of educational attainment. Try with both approaches, the classical and the  $p$ -value.

3. A random sample of 300 men and women were asked “How long should couples date before getting married?” Here are the results.

	Less Than 1 Year	1 Year	1-2 Years	2-3 Years	Longer Than 3 Years
Men	31	45	48	16	10
Women	29	43	51	16	11

At the 5% level of significance, test the claim that a person’s response to this question is independent of the person’s gender. Try with both approaches, the classical and the  $p$ -value.

4. The following table gives the distribution of grades for three professors for a few randomly selected classes that each of them taught during the past two years.

		Professor		
		Smith	Moore	McGregor
Grade	85-100	18	36	20
	70 – 84	25	44	15
	60 – 70	85	73	82
	Less than 60	17	12	8

Using a 2.5% significance level, can we conclude that the grades are independent of the professor? Try with both approaches, the classical and the p-value.

5. A sociologist conducted a survey to determine if the highest level of education attained by at least one of the partners is independent of or affects the number of years that a marriage will last (before ending in divorce). A summary of that survey is given below.

		Number of Years Marriage Lasted Before Ending in Divorce			
		0-1	2-5	6-15	16-20
Highest Education Level Attained by at Least One of the Partners	High School	91	82	74	40
	CEGEP	109	91	79	87
	University	133	111	96	68

Using a 5% level of significance, test the claim that the highest educational level attained by at least one of the partners is independent of the number of years that a marriage will last. Try with both approaches, the classical and the p-value.

6. The manager of an assembly process wants to determine whether the number of defective articles manufactured depends on the day of the week the articles are produced. She collected the following information.

Day of the Week	Monday	Tuesday	Wednesday	Thursday	Friday
Nondefective	85	90	95	95	90
Defective	15	10	5	5	10

Using a 10% level of significance, test the claim that the number of defective items is independent of the day of the week. Try with both approaches, the classical and the p-value.

## ANSWERS

- $H_0$ : Voting status is independent of age group.  
 $H_A$ : Voting status is dependent of age group.  
critical value:  $\chi^2_{(3,0.01)} = 11.34$       Reject  $H_0$   
test statistic:  $\chi^2 = 54.58$   
 $p$  – value  $< 0.005$
- $H_0$ : Smoking status is independent of educational attainment  
 $H_A$ : Smoking status is dependent of educational attainment  
critical value:  $\chi^2_{(3,0.05)} = 7.81$       Reject  $H_0$   
test statistic:  $\chi^2 = 12.73$   
 $0.005 < p$  – value  $< 0.01$
- $H_0$ : Responses are independent of gender.  
 $H_A$ : Responses are dependent of gender.  
critical value:  $\chi^2_{(4,0.05)} = 9.49$       Fail to reject  $H_0$   
test statistic:  $\chi^2 = 0.251$   
 $0.990 < p$  – value  $< 0.995$
- $H_0$ : Grade distribution is independent of the professor  
 $H_A$ : They are dependent  
critical value:  $\chi^2_{(6,0.025)} = 14.45$       Reject  $H_0$   
test statistic:  $\chi^2 = 21.58$   
 $p$  – value  $< 0.005$
- $H_0$ : The number of years that a marriage will last is independent of the highest educational level attained by at least one of the partners.  
 $H_A$ : They are dependent  
critical value:  $\chi^2_{(6,0.05)} = 12.59$       Fail to reject  $H_0$   
test statistic:  $\chi^2 = 12.10$   
 $0.05 < p$  – value  $< 0.1$
- $H_0$ : The number of defective items is independent of the day of the week  
 $H_A$ : They are dependent  
critical value:  $\chi^2_{(4,0.10)} = 7.78$       Reject  $H_0$   
test statistic:  $\chi^2 = 8.55$   
 $0.05 < p$  – value  $< 0.10$