MATHEMATICS 360-255-LW
Quantitative Methods II
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Fall 2009

XVI – Inferences for Independent Samples
SOLUTIONS

1. Two groups of subjects participated in an experiment designed to test the effect of frustration on aggression. The experimental group of 40 subjects received a frustrating puzzle to solve, whereas the control group of 40 subjects received an easy, nonfrustrating version of the same puzzle. Level of aggression was then measured for both groups. A mean aggression score of 4.0 was found for the experimental group (frustration), and of 3.0 for the control group (no frustration), where higher mean scores indicate greater aggression. Assume that the population standard deviation for the experimental group is 2.0, and 1.5 for the control group.
   a) Construct a 95% confidence interval for the difference in the mean aggression between the experimental group and the control group.

Step 1 Assumptions: $n_E = 40 \geq 30$, $n_C = 40 \geq 30$

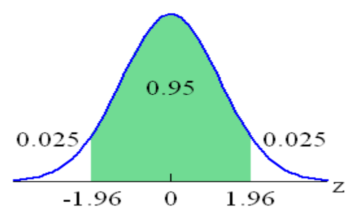
The samples are independent.

Step 2 a) Test statistic: z

b) Level of confidence: $1-\alpha = 0.95$ or $\alpha = 0.05$

Step 3 Point estimate: $\bar{x}_E - \bar{x}_C = 4.0 - 3.0 = 1.0$

Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$



b) $E = z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_B^2}{n_B} + \frac{\sigma_A^2}{n_A}} = 1.96\sqrt{\frac{2^2}{40} + \frac{1.5^2}{40}} = 0.775$

c) $(\bar{x}_E - \bar{x}_C) - E < \mu_E - \mu_C < (\bar{x}_E - \bar{x}_C) + E$

$1.0 - 0.775 < \mu_E - \mu_C < 1.0 + 0.775$

$0.225 < \mu_E - \mu_C < 1.775$

Step 5 The 95% confidence interval for the difference in the mean aggression between the experimental group and the control group is 0.225 to 1.775.

b) Test at the 5% significance level if frustration increases aggression. Try with both approaches, the classical and the *p*-value.

Classical Approach

Step 1 Assumptions: $n_E = 40 \geq 30$, $n_C = 40 \geq 30$

The samples are independent

Step 2 $H_O: \mu_E - \mu_C = 0$

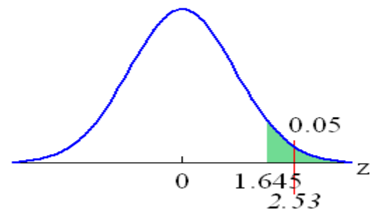
$H_A: \mu_E - \mu_C > 0$

Step 3 a) Test statistic: z

b) Right-tailed test with $\alpha = 0.05$

c) $z_\alpha = z_{0.05} = 1.645$

Step 4 $$z = \frac{(\bar{x}_E - \bar{x}_C) - (\mu_E - \mu_C)}{\sqrt{\frac{\sigma_E^2}{n_E} + \frac{\sigma_C^2}{n_C}}} = \frac{4.0 - 3.0}{\sqrt{\frac{2^2}{40} + \frac{1.5^2}{40}}} = 2.53$$



Step 5 a) z is in the critical region

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that frustration increases aggression.

p-value Approach

Step 1 Assumptions: $n_E = 40 \geq 30$, $n_C = 40 \geq 30$

The samples are independent

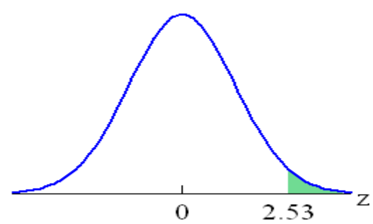
Step 2 $H_O: \mu_E - \mu_C = 0$

$H_A: \mu_E - \mu_C > 0$

Step 3 a) Test statistic: z

b) Right-tailed test with $\alpha = 0.05$

Step 4 a) $$z = \frac{(\bar{x}_E - \bar{x}_C) - (\mu_E - \mu_C)}{\sqrt{\frac{\sigma_E^2}{n_E} + \frac{\sigma_C^2}{n_C}}} = \frac{4.0 - 3.0}{\sqrt{\frac{2^2}{40} + \frac{1.5^2}{40}}} = 2.53$$



b) p-value = $P(z > 2.53) = 1 - 0.9943 = 0.0057$

Step 5 a) p-value = $0.0057 < \alpha = 0.05$

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that frustration increases aggression.

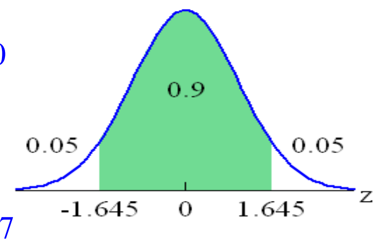
2. An experiment was conducted to compare the mean absorption of two drugs (A and B) in specimens of muscle tissue. Forty-eight tissue specimens were randomly divided into two equal groups. Each group was tested with one of the two drugs. The drug A had a mean absorption rate of 7.9, and drug B had a mean absorption rate of 8.5. Assume that absorption rates are normally distributed for both drugs, with a population standard deviation of 1.1 for drug A and of 1.2 for drug B.

a) Construct a 90% confidence interval for the difference in the mean absorption rates.

Step 1 Assumptions: Populations are normally distributed.
The samples are independent.

Step 2 a) Test statistic: z
b) Level of confidence: $1-\alpha = 0.90$ or $\alpha = 0.10$

Step 3 Point estimate: $\bar{x}_B - \bar{x}_A = 8.5 - 7.9 = 0.6$



Step 4 d) $z_{\frac{\alpha}{2}} = z_{0.05} = 1.645$

e) $E = z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_B^2}{n_B}+\frac{\sigma_A^2}{n_A}} = 1.645\sqrt{\frac{1.1^2}{24}+\frac{1.2^2}{24}} = 0.547$

f) $(\bar{x}_B - \bar{x}_A) - E < \mu_B - \mu_A < (\bar{x}_B - \bar{x}_A) + E$

$0.6 - 0.547 < \mu_B - \mu_A < 0.6 + 0.547$

$0.053 < \mu_B - \mu_A < 1.147$

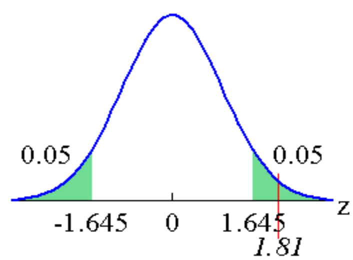
Step 5 The 90% confidence interval for the difference in the mean absorption rate is between the two drugs is 0.053 to 1.147.

b) Using a 10% level of significance, can you conclude that the absorption rate is different for the two drugs? Try with both approaches, the classical and the *p*-value.

Classical Approach

Step 1 Assumptions: Populations are normally distributed.
The samples are independent

Step 2 $H_O: \mu_B - \mu_A = 0$
$H_A: \mu_B - \mu_A \neq 0$



Step 3 a) Test statistic: z
b) Two-tailed test with $\alpha = 0.10$
c) $z_{\frac{\alpha}{2}} = z_{0.05} = 1.645$

Step 4 $z = \frac{(\bar{x}_B - \bar{x}_A) - (\mu_B - \mu_A)}{\sqrt{\frac{\sigma_B^2}{n_B}+\frac{\sigma_A^2}{n_A}}} = \frac{8.5-7.9}{\sqrt{\frac{1.1^2}{24}+\frac{1.2^2}{24}}} = 1.81$

Step 5 a) z is in the critical region
b) Reject H_o.
∴ There is sufficient evidence at the 10% level of significance to conclude that the mean absorption rate is different for the two drugs.

p-value Approach

Step 1 Assumptions: $n_A = 36 \geq 30$, $n_B = 36 \geq 30$

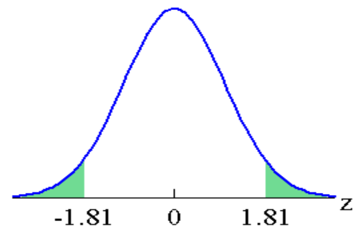
The samples are independent

Step 2 $H_O: \mu_B - \mu_A = 0$

$H_A: \mu_B - \mu_A \neq 0$

Step 3 a) Test statistic: z

b) Two-tailed test with $\alpha = 0.10$



Step 4 a) $z = \dfrac{(\bar{x}_B - \bar{x}_A) - (\mu_B - \mu_A)}{\sqrt{\dfrac{\sigma_B^2}{n_B} + \dfrac{\sigma_A^2}{n_A}}} = \dfrac{8.5 - 7.9}{\sqrt{\dfrac{1.1^2}{24} + \dfrac{1.2^2}{24}}} = 1.81$

b) p-value $= 2P(z > 1.81) = 2(1 - 0.9649) = 0.0702$

Step 5 a) p-value $= 0.0702 < \alpha = 0.10$

b) Reject H_o.

$\therefore$ There is sufficient evidence at the 10% level of significance to conclude that the mean absorption rate is different for the two drugs.

3. "Are intellectuals more likely to wear a beard?" wondered a social psychologist. To answer her question, she asked samples of bearded and clean-shaven men about their level of educational attainment. The following data on years of schooling was obtained.

Beard	18	23	12	14	18	16	15	19	12	21	19	8
	21	16	19	20	18	15	14	16	11	12	13	13
	14	19	18	23	19	17	17	12	11			
No Beard	16	11	12	15	14	12	11	19	11	22	13	9
	21	18	19	12	11	14	16	14	13	12	14	14
	12	11	11	15	17	18	21	22	23	14	13	

Assume that the population standard deviation for men with beards is 3.69 years and for men with no beards 3.77 years.

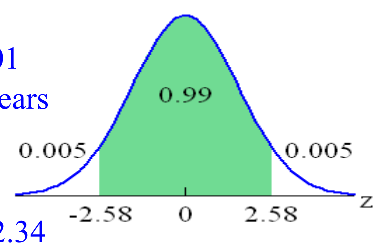
a) Construct a 99% confidence interval for the difference between the mean schooling of bearded and clean-shaven men.

Step 1 Assumptions: $n_b = 33 \geq 30$, $n_{nb} = 35 \geq 30$, the samples are independent.

Step 2 a) Test statistic: z

b) Level of confidence: $1 - \alpha = 0.99$ or $\alpha = 0.01$

Step 3 Point estimate: $\bar{x}_b - \bar{x}_{nb} = 16.15 - 14.86 = 1.29$ years



Step 4 g) $z_{\frac{\alpha}{2}} = z_{0.005} = 2.58$

h) $E = z_{\frac{\alpha}{2}} \sqrt{\dfrac{\sigma_b^2}{n_b} + \dfrac{\sigma_{nb}^2}{n_{nb}}} = 2.58 \sqrt{\dfrac{3.69^2}{33} + \dfrac{3.77^2}{35}} = 2.34$

i) $(\bar{x}_b - \bar{x}_{nb}) - E < \mu_b - \mu_{nb} < (\bar{x}_b - \bar{x}_{nb}) + E$

$1.29 - 2.34 < \mu_n - \mu_{nb} < 1.29 + 2.34$

$-1.05 < \mu_b - \mu_{nb} < 3.63$

Step 5 The 99% confidence interval for the difference in the mean schooling of bearded and clean-shaven men is -1.05 to 3.63 years.

b) Using a 1% level of significance, can you conclude bearded men have more schooling than clean-shaven men? Try with both approaches, the classical and the *p*-value.

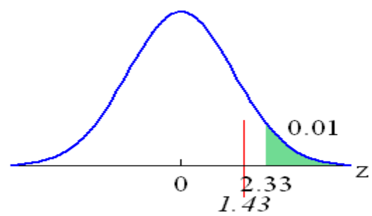
Classical Approach

Step 1 Assumptions: $n_b = 33 \geq 30$, $n_{nb} = 35 \geq 30$

The samples are independent

Step 2 $H_0 : \mu_b - \mu_{nb} = 0$

$H_A : \mu_b - \mu_{nb} > 0$



Step 3 a) Test statistic: z

b) Right-tailed test with α = 0.01

c) $z_\alpha = z_{0.01} = 2.33$

Step 4 $$z = \frac{(\bar{x}_b - \bar{x}_{nb}) - (\mu_b - \mu_{nb})}{\sqrt{\frac{\sigma_b^2}{n_b} + \frac{\sigma_{nb}^2}{n_{nb}}}} = \frac{16.15 - 14.86}{\sqrt{\frac{3.69^2}{33} + \frac{3.77^2}{35}}} = 1.43$$

Step 5 a) z is not in the critical region

b) Fail to reject H_o.

∴ There is not sufficient evidence at the 1% level of significance to conclude that the mean schooling of bearded men is higher than that of clean-shaven men.

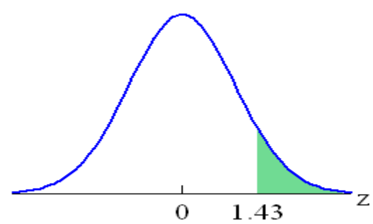
p-value Approach

Step 1 Assumptions: $n_b = 33 \geq 30$, $n_{nb} = 35 \geq 30$

The samples are independent

Step 2 $H_0 : \mu_b - \mu_{nb} = 0$

$H_A : \mu_b - \mu_{nb} > 0$



Step 3 a) Test statistic: z

b) Right-tailed test with α = 0.01

Step 4 a) $$z = \frac{(\bar{x}_b - \bar{x}_{nb}) - (\mu_b - \mu_{nb})}{\sqrt{\frac{\sigma_b^2}{n_b} + \frac{\sigma_{nb}^2}{n_{nb}}}} = \frac{16.15 - 14.86}{\sqrt{\frac{3.69^2}{33} + \frac{3.77^2}{35}}} = 1.43$$

b) *p*-value = $P(z > 1.43) = 1 - 0.9236 = 0.0764$

Step 5 a) *p*-value = 0.0764 > α = 0.01

b) Fail to reject H_o.

∴ There is not sufficient evidence at the 1% level of significance to conclude that the mean schooling of bearded men is higher than that of clean-shaven men.

4. The Thematic Apperception Test presents subjects with ambiguous pictures and asks them to tell a story about the picture. These stories can be scored in any number of ways. A psychologist asked mothers of 20 Normal and 20 Schizophrenic children to complete the TAT, and scored for the number of stories (out of 10) that exhibited a positive parent-child relationship. Assume TAT scores are normally distributed. Here are the results.

Normal	8	4	6	3	1	4	4	6	4	2
Schizophrenic	2	1	1	3	2	7	2	1	3	1

Normal	2	1	1	4	3	3	2	6	3	4
Schizophrenic	0	2	4	2	3	3	0	1	2	2

a) Construct a 95% confidence interval for the difference in the mean number of stories that exhibited a positive parent-child relationship between Normal and Schizophrenic children.

Step 1 Assumptions: Populations are normally distributed
The samples are independent.
Variances are equal.

Step 2 a) Test statistic: t with $df = n_N + n_S - 2 = 38$

b) Level of confidence: $1-\alpha = 0.95$ or $\alpha = 0.05$

Step 3 Point estimate: $\bar{x}_N - \bar{x}_S = 3.55 - 2.10 = 1.45$ stories

Step 4 a) $t_{\left(df,\frac{\alpha}{2}\right)} = t_{(38,0.025)} = 2.030$

b) $s_p = \sqrt{\dfrac{(n_N-1)s_N^2 + (n_S-1)s_S^2}{n_N+n_S-2}} = \sqrt{\dfrac{19\cdot 1.877^2 + 19\cdot 1.553^2}{38}} = 1.723$

$E = t_{\left(df,\frac{\alpha}{2}\right)} s_p \sqrt{\dfrac{1}{n_S}+\dfrac{1}{n_N}} = 2.030\cdot 1.723\sqrt{\dfrac{1}{20}+\dfrac{1}{20}} = 1.106$

a) $(\bar{x}_N - \bar{x}_S) - E < \mu_N - \mu_S < (\bar{x}_N - \bar{x}_S) + E$

$1.45 - 1.106 < \mu_N - \mu_S < 1.45 - 1.106$

$0.344 < \mu_N - \mu_S < 2.556$

Step 5 The 95% confidence interval for the difference in the mean number of stories that exhibited a positive parent-child relationship between Normal and Schizophrenic children is 0.344 to 2.556.

b) Using a 5% level of significance, can we conclude that normal children exhibit a more positive parent-child relationship? Try with both approaches, the classical and the *p*-value.

Classical Approach

Step 1 Assumptions: Populations are normally distributed
The samples are independent.
Variances are equal.

Step 2 $H_0 : \mu_N - \mu_S = 0$

$H_A : \mu_N - \mu_S > 0$

Step 3 a) Test statistic: t with $df = n_N + n_S - 2 = 38$

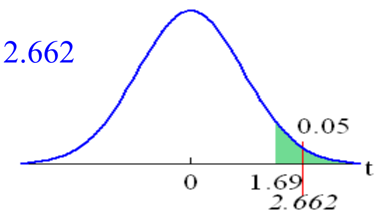
b) Right-tailed test with $\alpha = 0.05$

c) $t_{(df,\alpha)} = t_{(38,0.05)} = 1.690$

Step 4

$$s_p = \sqrt{\frac{(n_N - 1)s_N^2 + (n_S - 1)s_S^2}{n_N + n_S - 2}} = \sqrt{\frac{19 \cdot 1.877^2 + 19 \cdot 1.553^2}{38}} = 1.723$$

$$t = \frac{(\bar{x}_N - \bar{x}_S) - (\mu_N - \mu_S)}{s_p\sqrt{\frac{1}{n_N} + \frac{1}{n_S}}} = \frac{3.55 - 2.10}{1.723\sqrt{\frac{1}{20} + \frac{1}{20}}} = 2.662$$



Step 5 a) t is in the critical region

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that normal children exhibit a more positive parent-child relationship.

p-value Approach

Step 1 Assumptions: Populations are normally distributed
The samples are independent.
Variances are equal.

Step 2 $H_0 : \mu_N - \mu_S = 0$

$H_A : \mu_N - \mu_S > 0$

Step 3 a) Test statistic: t with $df = n_N + n_S - 2 = 38$

b) Two-tailed test with $\alpha = 0.05$

Step 4

$$s_p = \sqrt{\frac{(n_N - 1)s_N^2 + (n_S - 1)s_S^2}{n_N + n_S - 2}} = \sqrt{\frac{19 \cdot 1.877^2 + 19 \cdot 1.553^2}{38}} = 1.723$$

$$t = \frac{(\bar{x}_N - \bar{x}_S) - (\mu_N - \mu_S)}{s_p\sqrt{\frac{1}{n_N} + \frac{1}{n_S}}} = \frac{3.55 - 2.10}{1.723\sqrt{\frac{1}{20} + \frac{1}{20}}} = 2.662$$

$0.005 < p\text{-value} < 0.007$

Step 5 a) p-value $< 0.007 < \alpha = 0.05$

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that normal children exhibit a more positive parent-child relationship.

5. To study the influence of motivation on problem solving abilities, a psychologist asked 10 children to solve as many problems as they could in 10 minutes. One group (5 subjects) was told that this was a test of their innate problem-solving ability; a second group (5 subjects) was told that this was just a time-filling task. Assume the number of problems solved is normally distributed. Here are the results.

Innate ability	4	5	8	3	7
Time-filling task:	11	6	9	7	9

a) Construct a 99% confidence interval for the difference in the mean number of problems solved between the two experimental conditions.

Step 1 Assumptions: Populations are normally distributed
The samples are independent.
Variances are equal.

Step 2 a) Test statistic: t with $df = n_T + n_I - 2 = 8$

b) Level of confidence: $1-\alpha = 0.99$ or $\alpha = 0.01$

Step 3 Point estimate: $\bar{x}_T - \bar{x}_I = 8.4 - 5.3 = 3.0$ problems

Step 4 a) $t_{\left(df,\frac{\alpha}{2}\right)} = t_{(8,0.005)} = 3.355$

b) $s_p = \sqrt{\dfrac{(n_T-1)s_T^2 + (n_I-1)s_I^2}{n_T+n_I-2}} = \sqrt{\dfrac{4\cdot 1.949^2 + 4\cdot 2.074^2}{8}} = 2.012$

$E = t_{\left(df,\frac{\alpha}{2}\right)} s_p \sqrt{\dfrac{1}{n_T} + \dfrac{1}{n_I}} = 3.355 \cdot 2.012\sqrt{\dfrac{1}{5}+\dfrac{1}{5}} = 4.270$

c) $(\bar{x}_T - \bar{x}_I) - E < \mu_T - \mu_I < (\bar{x}_T - \bar{x}_I) + E$

$3.0 - 4.27 < \mu_T - \mu_I < 3.0 + 4.27$

$-1.27 < \mu_T - \mu_I < 7.27$

Step 5 The 95% confidence interval for the difference in the mean number of problems solved between the two experimental conditions is -1.27 to 7.27 problems.

b) Using a 1% level of significance, can we conclude that the mean number of problems solved vary with experimental condition? Try with both approaches, the classical and the *p*-value.

Classical Approach

Step 1 Assumptions: Populations are normally distributed
The samples are independent and the variances are equal.

Step 2 $H_0 : \mu_T - \mu_I = 0$

$H_A : \mu_T - \mu_I \neq 0$

Step 3 a) Test statistic: t with $df = n_T + n_I - 2 = 8$

b) Right-tailed test with $\alpha = 0.01$

c) $t_{\left(df, \frac{\alpha}{2}\right)} = t_{(8, 0.005)} = 3.355$

Step 4

$$s_p = \sqrt{\frac{(n_T - 1)s_T^2 + (n_I - 1)s_I^2}{n_T + n_I - 2}} = \sqrt{\frac{4 \cdot 1.949^2 + 4 \cdot 2.074^2}{8}} = 2.012$$

$$t = \frac{(\bar{x}_T - \bar{x}_I) - (\mu_T - \mu_I)}{s_p\sqrt{\frac{1}{n_T} + \frac{1}{n_I}}} = \frac{8.4 - 5.4}{2.012\sqrt{\frac{1}{5} + \frac{1}{5}}} = 2.357$$

Step 5 a) t is not in the critical region

b) Fail to reject H_0.

∴ There is not sufficient evidence at the 1% level of significance to conclude that the mean number of problems solved varies with experimental condition.

p-value Approach

Step 1 Assumptions: Populations are normally distributed
The samples are independent and the variances are equal.

Step 2 $H_0 : \mu_T - \mu_I = 0$

$H_A : \mu_T - \mu_I \neq 0$

Step 3 a) Test statistic: t with $df = n_T + n_I - 2 = 8$

b) Two-tailed test with $\alpha = 0.01$

Step 4

$$s_p = \sqrt{\frac{(n_T - 1)s_T^2 + (n_I - 1)s_I^2}{n_T + n_I - 2}} = \sqrt{\frac{4 \cdot 1.949^2 + 4 \cdot 2.074^2}{8}} = 2.012$$

$$t = \frac{(\bar{x}_T - \bar{x}_I) - (\mu_T - \mu_I)}{s_p\sqrt{\frac{1}{n_T} + \frac{1}{n_I}}} = \frac{8.4 - 5.4}{2.012\sqrt{\frac{1}{5} + \frac{1}{5}}} = 2.357$$

$$2 \cdot 0.022 < p\text{-value} < 2 \cdot 0.025$$

$$0.044 < p\text{-value} < 0.050$$

Step 5 a) $p\text{-value} > 0.044 > \alpha = 0.01$

b) Fail to reject H_0.

∴ There is not sufficient evidence at the 1% level of significance to conclude that the mean number of problems solved varies with experimental condition.

6. A criminologist was interested in determining whether there was disparity in sentencing based on the race of the defendant. She selected at random 18 burglary convictions and compared the prison terms given to the 10 whites and 8 blacks sampled. Assume the length of sentences is normally distributed. The sentence lengths (in years) are shown for the white and black offenders.

Whites	3	5	4	7	4	5	6	4	3	2
Blacks	4	8	7	3	5	4	5	4		

a) Construct a 90% confidence interval for the difference in the mean sentence length between blacks and whites convicted of burglary.

Step 1 Assumptions: Populations are normally distributed
The samples are independent.
Variances are equal.

Step 2 a) Test statistic: t with $df = n_B + n_W - 2 = 16$

b) Level of confidence: $1-\alpha = 0.90$ or $\alpha = 0.10$

Step 3 Point estimate: $\bar{x}_B - \bar{x}_W = 5.0 - 4.3 = 0.7$ years

Step 4 a) $t_{\left(df,\frac{\alpha}{2}\right)} = t_{(16,0.05)} = 1.746$

b) $$s_p = \sqrt{\frac{\left(n_B - 1\right)s_B^2 + \left(n_W - 1\right)s_W^2}{n_B + n_W - 2}} = \sqrt{\frac{7 \cdot 1.690^2 + 9 \cdot 1.494^2}{16}} = 1.583$$

$$E = t_{\left(df,\frac{\alpha}{2}\right)} s_p \sqrt{\frac{1}{n_B} + \frac{1}{n_W}} = 1.746 \cdot 1.583 \sqrt{\frac{1}{8} + \frac{1}{10}} = 1.311$$

c) $$\left(\bar{x}_B - \bar{x}_W\right) - E < \mu_B - \mu_W < \left(\bar{x}_B - \bar{x}_W\right) + E$$

$$0.7 - 1.311 < \mu_B - \mu_W < 0.7 + 1.311$$

$$-0.611 < \mu_B - \mu_W < 2.011$$

Step 5 The 90% confidence interval for the difference in the mean sentence length between blacks and whites convicted of burglary is -0.611 to 2.011 years.

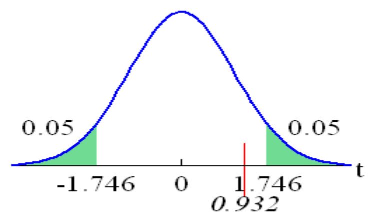
b) Using a 10% level of significance, can we conclude that whites and blacks convicted of burglary in this jurisdiction do no differ with respect to prison sentence length? Try with both approaches, the classical and the *p*-value.

Classical Approach

Step 1 Assumptions: Populations are normally distributed
The samples are independent and the variances are equal.

Step 2 $H_0 : \mu_B - \mu_W = 0$

$H_A : \mu_B - \mu_W \neq 0$



Step 3 a) Test statistic: t with $df = n_B + n_W - 2 = 16$

b) Right-tailed test with α = 0.10

c) $t_{(df,\frac{\alpha}{2})} = t_{(16,0.05)} = 1.746$

Step 4

$$s_p = \sqrt{\frac{(n_B - 1)s_B^2 + (n_W - 1)s_W^2}{n_B + n_W - 2}} = \sqrt{\frac{7\cdot 1.690^2 + 9\cdot 1.494^2}{16}} = 1.583$$

$$t = \frac{(\bar{x}_B - \bar{x}_W) - (\mu_B - \mu_W)}{s_p\sqrt{\frac{1}{n_B} + \frac{1}{n_W}}} = \frac{5.0 - 4.3}{1.583\sqrt{\frac{1}{8} + \frac{1}{10}}} = 0.932$$

Step 5 a) t is not in the critical region

b) Fail to reject H_o.

∴ There is not sufficient evidence at the 1% level of significance to conclude that whites and blacks convicted of burglary in this jurisdiction differ with respect to prison sentence length.

p-value Approach

Step 1 Assumptions: Populations are normally distributed
The samples are independent and the variances are equal.

Step 2 $H_0 : \mu_B - \mu_W = 0$

$H_A : \mu_B - \mu_W \neq 0$

Step 3 a) Test statistic: t with $df = n_B + n_W - 2 = 16$

b) Two-tailed test with α = 0.10

Step 4

$$s_p = \sqrt{\frac{(n_B - 1)s_B^2 + (n_W - 1)s_W^2}{n_B + n_W - 2}} = \sqrt{\frac{7\cdot 1.690^2 + 9\cdot 1.494^2}{16}} = 1.583$$

$$t = \frac{(\bar{x}_B - \bar{x}_W) - (\mu_B - \mu_W)}{s_p\sqrt{\frac{1}{n_B} + \frac{1}{n_W}}} = \frac{5.0 - 4.3}{1.583\sqrt{\frac{1}{8} + \frac{1}{10}}} = 0.932$$

$$2\cdot 0.165 < p\text{-value} < 2\cdot 0.191$$

$$0.330 < p\text{-value} < 0.382$$

Step 5 a) p-value > 0.330 > α = 0.10

b) Fail to reject H_o.

∴ There is not sufficient evidence at the 1% level of significance to conclude that whites and blacks convicted of burglary in this jurisdiction differ with respect to prison sentence length.