MATHEMATICS 360-255-LW
Quantitative Methods II
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Fall 2009

XIV – Hypothesis Testing for Proportions
SOLUTIONS

1. It is claimed that 60% of all 18-25-year-olds have used alcohol in the past 30 days. A survey of 125 students on campus who are between the ages of 18 and 25 showed that 83 have used alcohol in the past 30 days. Test the claim at the 5% level of significance and use the classical approach.

Step 1 Assumptions: $n = 125 > 20$ $np = 125 \cdot 0.6 = 75 > 5$ and $nq = 50 > 5$

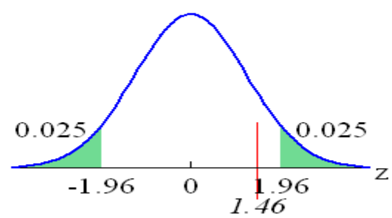
Step 2 $H_0 : p = 0.6$

$H_A : p \neq 0.6$

Step 3 a) Test statistic: z

b) Two-tailed test with $\alpha = 0.05$

c) $z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$



Step 4 $$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{83}{125} - 0.6}{\sqrt{\frac{0.6 \cdot 0.4}{125}}} = 1.46$$

Step 5 a) z is not in the critical region.

b) Fail to reject H_0.

$\therefore$ There is not sufficient evidence at the 5% level of significance to conclude that the proportion of 18-25-year-olds who have used alcohol in the past 30 days is different than 60%.

2. The author of a blackjack strategy book claims that the dealer wins 52.5% of all hands dealt. A blackjack player played 100 hands against a computer dealer, and the computer dealer won 58 of the hands. Use these data to test the author's claim at the 5% level of significance. Use the p-value approach.

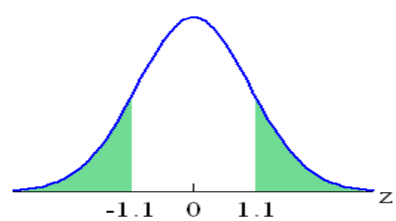
Step 1 Assumptions: $n = 100 > 20$ $np = 52.2 > 5$ and $nq = 47.5 > 5$

Step 2 $H_0 : p = 0.525$

$H_A : p \neq 0.525$

Step 3 a) Test statistic: z

b) Two-tailed test with $\alpha = 0.05$



Step 4 a) $$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{58}{100} - 0.525}{\sqrt{\frac{0.525 \cdot 0.475}{100}}} = 1.10$$

b) p-value = $2P(z < -1.10) = 2 \cdot 0.1357 = 0.2714$

Step 5 a) p-value $= 0.2714 > \alpha = 0.05$

b) Fail to reject H_0.

$\therefore$ There is not sufficient evidence at the 2% level of significance to reject the author's claim.

3. It has been claimed that at least 2 out of every 3 pet owners allow their pets to sleep in bed with them. A survey of 295 pet owners at a local pet shop showed that 236 of them allowed their pet to sleep in bed with them. At the 5% level of significance, test the claim that at least 2/3 of all pet owners allow their pets to sleep in bed with them.

a) Use the classical approach.

Step 1 Assumptions: $n = 295 > 20$ $np = 295 \cdot \frac{2}{3} = 196.7 > 5$

$nq = 94.3 > 5$

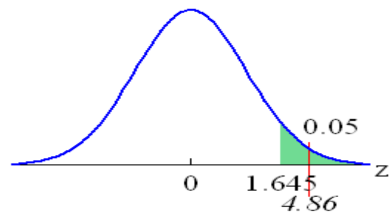
Step 2 $H_0 : p = \frac{2}{3}$

$H_A : p > \frac{2}{3}$

Step 3 a) Test statistic: z

b) Right-tailed test with α = 0.05

c) $z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$



Step 4 $$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{236}{295} - \frac{2}{3}}{\sqrt{\frac{\frac{2}{3}\frac{1}{3}}{295}}} = 4.86$$

Step 5 a) z is in the critical region.

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that at least 2/3 of all pet owners allow their pets to sleep in bed with them.

b) Use the p-value approach.

Step 1 Assumptions: $n = 295 > 20$ $np = 295 \cdot \frac{2}{3} = 196.7 > 5$

$nq = 94.3 > 5$

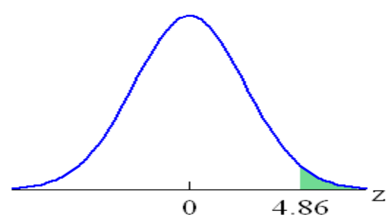
Step 2 $H_0 : p = \frac{2}{3}$

$H_A : p > \frac{2}{3}$

Step 3 a) Test statistic: z

b) Right-tailed test with α = 0.05

Step 4 a) $$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{236}{295} - \frac{2}{3}}{\sqrt{\frac{\frac{2}{3}\frac{1}{3}}{295}}} = 4.86$$



b) p-value = $P(z > 4.86) = 1 - 1.000 = 0.000$

Step 5 a) p-value = 0.000 < α = 0.05

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that at least 2/3 of all pet owners allow their pets to sleep in bed with them.

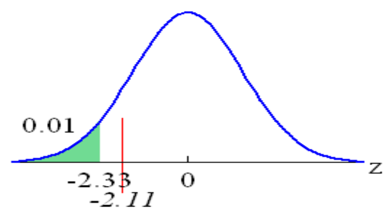
4. A random sample of 225 adult Canadians showed that 13 of them had more than one job. At the 1% level of significance, test the claim that that less than 10% of all adult Canadians have more than one job.

a) Use the classical approach.

Step 1 Assumptions: $n = 225 > 20$ $np = 225 \cdot 0.1 = 22.5 > 5$
$nq = 202.5 > 5$

Step 2 $H_0 : p = 0.10$
$H_A : p < 0.10$

Step 3 a) Test statistic: z
b) Left-tailed test with $\alpha = 0.01$
c) $z_\alpha = z_{0.01} = 2.33$



Step 4 $z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{13}{225} - 0.1}{\sqrt{\frac{0.1 \cdot 0.9}{225}}} = -2.11$

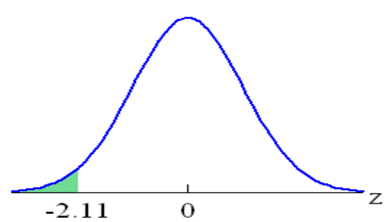
Step 5 a) z is not in the critical region.
b) Fail to reject H_o.
$\therefore$ There is not sufficient evidence at the 5% level of significance to conclude that less than 10% of all adult Canadians have more than one job.

b) Use the p-value approach.

Step 1 Assumptions: $n = 225 > 20$ $np = 225 \cdot 0.1 = 22.5 > 5$
$nq = 202.5 > 5$

Step 2 $H_0 : p = 0.10$
$H_A : p < 0.10$

Step 3 a) Test statistic: z
b) Left-tailed test with $\alpha = 0.01$



Step 4 a) $z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{13}{225} - 0.1}{\sqrt{\frac{0.1 \cdot 0.9}{225}}} = -2.11$
b) p-value = $P(z < -2.11) = 0.2946$

Step 5 a) p-value = $0.2946 > \alpha = 0.01$
b) Fail to reject H_o..
$\therefore$ There is not sufficient evidence at the 5% level of significance to conclude that less than 10% of all adult Canadians have more than one job.

5. A random sample of 118 adult women revealed that 70 were satisfied with their weight. At the 2% level of significance, test the claim that more than half of all adult women are satisfied with their weight.

   a) Use the classical approach.

Step 1 Assumptions: $n = 118 > 20 \quad np = 118 \cdot \frac{1}{2} = 59 > 5 \quad nq = 59 > 5$

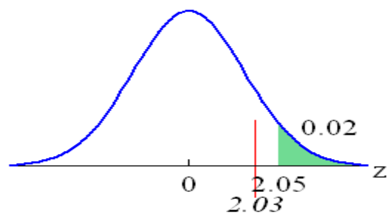
Step 2 $H_0 : p = 0.5$

$H_A : p > 0.5$

Step 3 a) Test statistic: z

b) Right-tailed test with $\alpha = 0.02$

c) $z_\alpha = z_{0.02} = 2.05$



Step 4 $$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{70}{118} - \frac{1}{2}}{\sqrt{\frac{\frac{1}{2}\frac{1}{2}}{118}}} = 2.03$$

Step 5 a) z is not in the critical region.

b) Fail to reject H_0.

$\therefore$ There is not sufficient evidence at the 2% level of significance to conclude that more than half of all adult women are satisfied with their weight.

   b) Use the p-value approach.

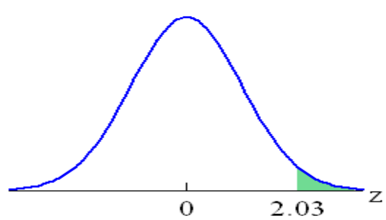
Step 1 Assumptions: $n = 118 > 20 \quad np = 118 \cdot \frac{1}{2} = 59 > 5 \quad nq = 59 > 5$

Step 2 $H_0 : p = 0.5$

$H_A : p > 0.5$

Step 3 a) Test statistic: z

b) Right-tailed test with $\alpha = 0.02$



Step 4 a) $$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{70}{118} - \frac{1}{2}}{\sqrt{\frac{\frac{1}{2}\frac{1}{2}}{118}}} = 2.03$$

b) p-value = $P(z > 2.03) = 1 - 0.9788 = 0.0212$

Step 5 a) p-value = $0.0212 > \alpha = 0.02$

b) Fail to reject H_0..

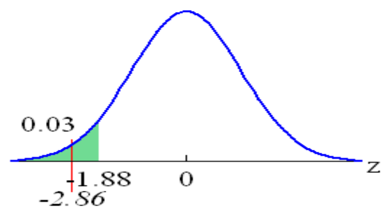
$\therefore$ There is not sufficient evidence at the 2% level of significance to conclude that more than half of all adult women are satisfied with their weight.

6. A mail-order company claims that 60% of all orders are mailed within 48 hours. From time to time the quality control department at the company checks if this promise is fulfilled. Recently the quality control department at this company took a sample of 400 orders and found that 212 of them were mailed within 48 hours of the placement of the orders. Testing at the 3% significance level, can you conclude that less than 60% of all orders are mailed within 48 hours?

   a) Use the classical approach.

   Step 1 Assumptions: $n = 400 > 20$ $np = 400 \cdot 0.6 = 240 > 5$
   $nq = 160 > 5$

   Step 2 $H_0 : p = 0.6$
   $H_A : p < 0.6$

   Step 3 a) Test statistic: z
   b) Left-tailed test with $\alpha = 0.03$
   c) $z_\alpha = z_{0.03} = 1.88$

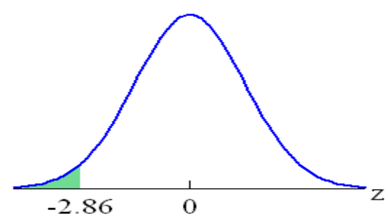
   Step 4 $$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{212}{400} - 0.6}{\sqrt{\frac{0.6 \cdot 0.4}{400}}} = -2.86$$

   Step 5 a) z is in the critical region.
   b) Reject H_o.
   ∴ There is sufficient evidence at the 5% level of significance to conclude that that less than 60% of all orders are mailed within 48 hours.

   b) Use the p-value approach.

   Step 1 Assumptions: $n = 295 > 20$ $np = 295 \cdot \frac{2}{3} = 196.7 > 5$
   $nq = 94.3 > 5$

   Step 2 $H_0 : p = 0.6$
   $H_A : p < 0.6$

   Step 3 a) Test statistic: z
   b) Left-tailed test with $\alpha = 0.03$

   Step 4 a) $$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{212}{400} - 0.6}{\sqrt{\frac{0.6 \cdot 0.4}{400}}} = -2.86$$
   b) p-value = $P(z < -2.86) = 0.0021$

   Step 5 a) p-value = $0.0021 < \alpha = 0.03$
   b) Reject H_o.
   ∴ There is sufficient evidence at the 5% level of significance to conclude that that less than 60% of all orders are mailed within 48 hours.

   c) What is a type I error? What is the probability of making a type I error?

   A type I error in this case is rejecting H_0 when H_0 is in fact true. That is, if p = 0.60 is in fact true, but we conclude that it is not. The probability of a Type I error is 0.03.

7. What is wrong with this statistical project?
   A student in QM II wants to test the claim that more than half of all CEGEP students live at home with their parents. To prove her point, she takes a random sample of 75 SLC students, and finds that 48 live at home with their parents. She then uses the hypothesis test introduced for this section and concludes that more than half of CEGEP students live at home with their parents.

   The only conclusion the student can make is that more than half of SLC students live at home with their parents. The generalization to all CEGEP students does not hold because the sample is not representative of all CEGEP students since it was all taken from St. Lawrence.

8. What is wrong with applying the hypothesis test of this section to the results of a Web Poll?

   The sample is not random, so the results cannot be generalized. Web polls are meaningless.