MATHEMATICS 360-255-LW
Quantitative Methods II
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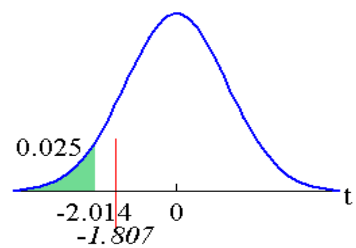
XIII – Hypothesis Testing for μ (unknown σ)
SOLUTIONS

1. T The dean of a university claims that the mean time spent partying by all students at this university is less than 7 hours per week. A random sample of 48 students taken from this university showed that they spent an average of 6.4 hours partying the previous week with a standard deviation of 2.3 hours. Can you conclude, at the 2.5% significance level, that the president's claim is true? Use the classical approach.

Step 1 Assumptions: $n = 48 \geq 30$

Step 2 $H_0 : \mu = 7$ hours

$H_A : \mu < 7$ hours



Step 3 a) Test statistic: t with $df = 47$

b) Left-tailed test with α = 0.025

c) $t_{(df,1-\alpha)} = t_{(19,0.975)} = -2.014$

Step 4 $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{6.4 - 7}{\frac{2.3}{\sqrt{48}}} = -1.807$

Step 5 a) t is not in the critical region

b) Fail to reject H_o.

∴ There is not sufficient evidence at the 5% level of significance to conclude that the dean's claim is true.

2. A past study claims that adult Canadians spend an average of 18 hours a week on leisure activities. A researcher wanted to test this claim. She took a sample of 10 adults and asked them about the time they spend per week on leisure activities. Their responses (in hours) are as follows.

14	25	22	38	16	26	19	23	41	33

Assume that the time spent on leisure activities by all adults is normally distributed. Using the 5% significance level, can you conclude that the claim of the earlier study is true? Use the *p*-value approach.

Step 1 Assumptions: Population is normally distributed

Step 2 $H_0 : \mu = 18$ hours

$H_A : \mu \neq 18$ hours

Step 3 a) Test statistic: t with $df = 9$

b) Two-tailed test with $\alpha = 0.05$

Step 4 a) $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{25.7 - 18}{\frac{9.04}{\sqrt{10}}} = 2.692$

b) $2 \cdot 0.011 < p - \text{value} < 2 \cdot 0.016$

$0.022 < p - \text{value} < 0.032$

Step 5 a) p-value $< 0.032 < \alpha = 0.05$

b) Reject H_0.

∴ There is not sufficient evidence at the 5% level of significance to conclude that the earlier claim is false.

3. How many pair of shoes do female college students own? A random sample of 15 female college students produced a sample mean of 8.7 pairs of shoes, with a standard deviation of 0.85 pairs. Use these data to test the claim that the mean number of pairs of shoes owned by female college students is less than 10 at the 5% level of significance. Assume that the number of pair of shoes owned by female students is normally distributed.

   a) Use the classical approach.

      Step 1 Assumptions: Population is normally distributed

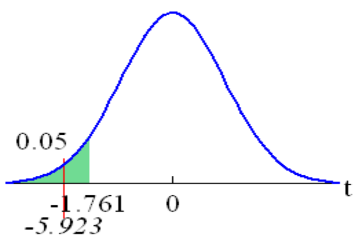
      Step 2 $H_0 : \mu = 10$ pairs of shoes

      $H_A : \mu < 10$ pairs of shoes

      Step 3 a) Test statistic: t with $df = 14$

      b) Left-tailed test with $\alpha = 0.05$

      c) $t_{(df,1-\alpha)} = t_{(9,0.95)} = -1.761$



      Step 4 $t = \dfrac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \dfrac{8.7-10}{\frac{0.85}{\sqrt{15}}} = -5.923$

      Step 5 a) t is in the critical region

      b) Reject H_o.

      ∴ There is sufficient evidence at the 5% level of significance to conclude that the mean number of pairs of shoes owned by female college students is less than 10.

   b) Use the *p*-value approach.

      Step 1 Assumptions: Population is normally distributed

      Step 2 $H_0 : \mu = 10$ pairs of shoes

      $H_A : \mu < 10$ pairs of shoes

      Step 3 a) Test statistic: t with $df = 14$

      b) Two-tailed test with $\alpha = 0.05$

      Step 4 a) $t = \dfrac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \dfrac{8.7-10}{\frac{0.85}{\sqrt{15}}} = -5.923$

      b) $p - \text{value} < 0.001$

      Step 5 a) p-value $< 0.001 < \alpha = 0.05$

      b) Reject H_o.

      ∴ There is sufficient evidence at the 5% level of significance to conclude that the mean number of pairs of shoes owned by female college students is less than 10.

4. These are the number of packages handled by a shipping office on 17 randomly selected days.

1103	1488	1713	1536	1037	1462	1625	1627	1547
1080	1216	1639	1539	1545	907	1307	1387	

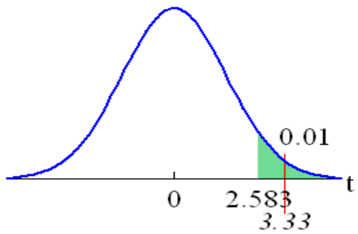
Test the claim that the shipping office handles more than 1200 packages per day at the 1% level of significance. Assume that the number of packages handled by a shipping office in a day is normally distributed.

a) Use the classical approach.

Step 1 Assumptions: Population is normally distributed

Step 2 $H_0 : \mu = 1200$ packages

$H_A : \mu > 1200$ packages



Step 3 a) Test statistic: t with $df = 16$

b) Right-tailed test with $\alpha = 0.01$

c) $t_{(df,\alpha)} = t_{(16,0.01)} = 2.583$

Step 4 $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{1397.5 - 1200}{\frac{244.60}{\sqrt{17}}} = 3.330$

Step 5 a) t is in the critical region

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that the shipping office handles more than 1200 packages per day.

b) Use the p-value approach.

Step 1 Assumptions: Population is normally distributed

Step 2 $H_0 : \mu = 1200$ packages

$H_A : \mu > 1200$ packages

Step 3 a) Test statistic: t with $df = 16$

b) Right-tailed test with $\alpha = 0.01$

Step 4 a) $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{1397.5 - 1200}{\frac{244.60}{\sqrt{17}}} = 3.330$

b) p-value = 0.002

Step 5 a) p-value = 0.002 < α = 0.01

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that the shipping office handles more than 1200 packages per day.

5. Major league baseball managers keep a close eye on the "pitch count" of their starting pitcher, because they believe that a pitcher loses his effectiveness after a certain number of pitches. Here are the pitch counts of 16 randomly selected starting pitchers.

117	94	137	66	103	100	81	98
86	77	94	95	115	79	93	86

At the 5% level of significance, test the claim that the mean pitch count for starting pitchers is 100 pitches. Assume that the pitch count for starting pitchers is normally distributed.

a) Use the classical approach.

Step 1 Assumptions: Population is normally distributed

Step 2 $H_0 : \mu = 100$ pitches

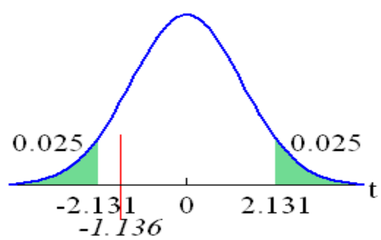
$H_A : \mu \neq 100$ pitches

Step 3 a) Test statistic: t with $df = 15$

b) Two-tailed test with $\alpha = 0.05$

c) $t_{\left(df, \frac{\alpha}{2}\right)} = t_{(15, 0.025)} = 2.131$

Step 4 $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{95.1 - 100}{\frac{17.38}{\sqrt{16}}} = -1.136$



Step 5 a) t is not in the critical region

b) Fail to reject H_o.

∴ There is not sufficient evidence at the 5% level of significance to conclude that the mean pitch count for starting pitchers is not 100 pitches.

b) Use the p-value approach.

Step 1 Assumptions: Population is normally distributed

Step 2 $H_0 : \mu = 100$ pitches

$H_A : \mu \neq 100$ pitches

Step 3 a) Test statistic: t with $df = 15$

b) Two-tailed test with $\alpha = 0.05$

Step 4 a) $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{95.1 - 100}{\frac{17.38}{\sqrt{16}}} = -1.136$

b) $2 \cdot 0.124 < p - \text{value} < 2 \cdot 0.144$

$0.248 < p - \text{value} < 0.288$

Step 5 a) p-value $> 0.248 > \alpha = 0.05$

b) Fail to reject H_o.

∴ There is not sufficient evidence at the 5% level of significance to conclude that the mean pitch count for starting pitchers is not 100 pitches.

6. A random sample of 85 workers was taken, where each was asked for how long they browsed the net for personal use while at work. The mean length found was 97.4 minutes with a standard deviation of 41.3 minutes. Use this sample to test the claim, at the 5% level of significance, that the mean time spent browsing the net for personal use while at work is greater than 90 minutes.

a) Use the classical approach.

Step 1 Assumptions: $n = 85 \geq 30$

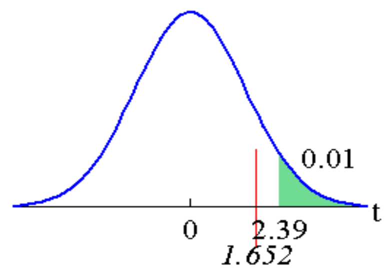
Step 2 $H_0 : \mu = 90$ min

$H_A : \mu > 90$ min

Step 3 a) Test statistic: t with $df = 84$

b) Right-tailed test with $\alpha = 0.01$

c) $t_{(df,\alpha)} = t_{(84,0.01)} = 2.390$



Step 4 $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{97.4 - 90}{\frac{41.3}{\sqrt{85}}} = 1.652$

Step 5 a) t is not in the critical region

b) Fail to reject H_o.

∴ There is not sufficient evidence at the 5% level of significance to conclude that the mean time spent browsing the net for personal use while at work is greater than 90 minutes.

b) Use the p-value approach.

Step 1 Assumptions: $n = 85 \geq 30$

Step 2 $H_0 : \mu = 90$ min

$H_A : \mu > 90$ min

Step 3 a) Test statistic: t with $df = 84$

b) Right-tailed test with $\alpha = 0.01$

Step 4 a) $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{97.4 - 90}{\frac{41.3}{\sqrt{85}}} = 1.652$

b) $0.048 < p - \text{value} < 0.058$

Step 5 a) p-value $> 0.048 > \alpha = 0.01$

b) Fail to reject H_o.

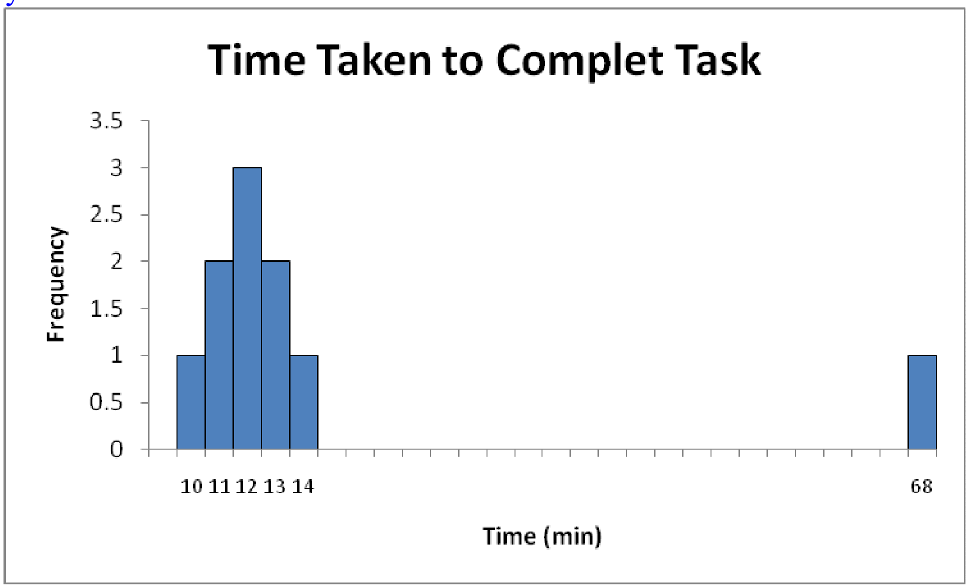
∴ There is not sufficient evidence at the 5% level of significance to conclude that the mean time spent browsing the net for personal use while at work is greater than 90 minutes.

7. What is wrong with this statistical project?
A student in QM II randomly selects 10 students at SLC, and asks each of them to note the time (in min) it takes them to complete a particular task. Here are the results

10 11 11 12 12 12 13 13 14 68

She then uses these results to claim that the mean time taken to complete the test is over 10 minutes, using the hypothesis test introduced for this section.

Because of the number 68, the assumption that the population is normally distributed is probably false.



Numbers, such as this 68, that are not consistent with the bulk of the data are called **outliers**. Under certain circumstances, they can be removed and the hypothesis test conducted with the remaining data.