MATHEMATICS 360-255-LW
Quantitative Methods II
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Fall 2009

XII – Hypothesis Testing for μ (known σ)

1. Write the null and alternative hypotheses for each of the following examples. Determine if each is a case of a two-tailed, a left-tailed, or a right-tailed test.
   a) To test whether or not the mean price of houses in Quebec is greater than $143000.
   b) To test if the mean number of hours spent working per week by college students who hold jobs is different from 15 hours.
   c) To test whether the mean life of a particular brand of auto batteries is less than 45 months.
   d) To test if the mean amount of time spent doing homework by all fourth-graders is different from 5 hours a week.
   e) To test if the mean age of all college students is different from 18 years.

2. Consider $H_0 : \mu = 20$ versus $H_A : \mu < 20$
   a) What type of error would you make if the null hypothesis is actually false and you fail to reject it?
   b) What type of error would you make if the null hypothesis is actually true and you reject it?

3. According to a Pharmacist, Quebecers spent an average of $220 per person on prescription drugs in 1998. A recent survey of 300 randomly chosen Quebecers showed that they spent an average of $235 per prescription drugs. Test at the 2.5% significance level whether the mean amount currently spent on prescription drugs by all Quebecers exceed $220 per person, if previous research has shown that the population a standard deviation is $90. Use the classical approach.

4. According to *Statistics Canada*, the average family income in Canada was $76 100 in 2004. A recently taken sample of 1200 Canadian families yielded a mean income of $77 152. Using the 2% significance level, can you conclude that the mean family income in Canada has changed since 2004? Assume $\sigma =$ $16 850. Use the classical approach.

5. A study conducted a few years ago claims that adult males spend an average of 11 hours a week watching sports on television. A recent sample of 29 adult males showed that the mean time they spend per week watching television is 9.50 hours. Test at the 1% significance level if currently all adult males spend less than 11 hours watching sports on television. Assume that the number of hours adult males spend watching TV is normally distributed with a population standard deviation of 2.2 hours. Use the classical approach.

6. A telephone company claims that the mean duration of all long-distance phone calls made by its residential customers is 10 minutes. A random of 21 long-distance calls made by its residential customers taken from the records of this company showed that the mean duration of calls for this sample is 8.2 minutes. Assuming the duration of long-distance calls is normally distributed, test the whether the mean duration of all long-distance calls is less than 10 minutes. (Use $\sigma = 4.6$ minutes).
   (a) at the 2% level of significance
   (b) at the 5% level of significance
   Use the *p*-value approach.

7. For a population of humans to sustain itself, there must be an average of just of over2 births for each woman of reproductive age. The fertility rate varies substantially from within one nation to the next. A journalist recently pointed out in *Fortune* magazine that the average for Japan has dropped to 1.5 births for each woman of reproductive age, which could reduce Japan's population from 135 million in 1997 to 50 million by the end of the 21st century. Suppose a random sample of 200 Japanese women of reproductive age is taken in 2008, and the sample mean fertility rate is measured as 1.45. Did the rate decline? Assume a population standard deviation of 0.75 births. Use a 5% level of significance, with
   a) the classical approach
   b) the *p*-value approach

8. A politician claims that the mean number of hours Canadians worked per week in a typical week is greater than 40 hours. A sample of 17 Canadians workers produced a mean of 44.7 hours per week. Assuming the number of hours Canadians work per week is normally distributed, and that previous research has shown that the population standard deviation is 10.63 hours per week, test the politician's claim at the 5% level of significance using
   a) the classical approach
   b) the *p*-value approach

9. The customers at a bank complained about long lines and the time they had to spend waiting for service. It is known that the customers at this bank had to wait 8 minutes, on average, before being served. The management made some changes to reduce the waiting time for its customers. A sample of 32 customers taken after these changes were made produced a mean waiting time of 7.4 minutes. Using this sample mean, the bank manager displayed a huge banner inside the bank mentioning that the mean waiting time for customer has been reduced by new changes. Do you think the manager's claim is justifiable? Use a 2.5% level of significance along with the classical approach, assuming $\sigma = 2.1$ minutes.

10. A sample of 26 parents was taken, where each was asked for how the amount of time they spent per week on school work or school-related activities. This sample produced a mean of 6.3 hours per week. Assume the amount of time they spent per week on school work or school-related activities is normally distributed with a population standard deviation of 4.4 hours. At the 0.01 level of significance, test the claim that the mean number of hours spent by parents on school work or school-related activities is 5 hours per week, using
    a) the classical approach
    b) the *p*-value approach

ANSWERS

1. a) $H_0 : \mu = \$143000$ Right-tailed
   $H_A : \mu > \$143000$
   b) $H_0 : \mu = 15$ hours Two-tailed
   $H_A : \mu \neq 15$ hours
   c) $H_0 : \mu = 45$ months Left-tailed
   $H_A : \mu < 45$ months
   d) $H_0 : \mu = 5$ hours Two-tailed
   $H_A : \mu \neq 5$ hours
   e) $H_0 : \mu = 18$ years Two-tailed
   $H_A : \mu \neq 18$ years
2. a) Type II error b) Type I error
3. $H_0 : \mu = \$220$ critical value: $z_{0.025} = 1.96$ Reject H_0
   $H_A : \mu > \$220$ test statistic: $z = 2.89$
4. $H_0 : \mu = \$76\ 100$ critical values: $\pm z_{0.01} = \pm 2.33$ Fail to reject H_0
   $H_A : \mu \neq \$76\ 100$ test statistic: $z = 2.16$
5. $H_0 : \mu = 11$ hours critical value: $-z_{0.025} = -2.33$ Reject H_0
   $H_A : \mu < 11$ hours test statistic: $z = -3.67$
6. $H_0 : \mu = 10$ minutes test statistic: $z = -1.79$ a) Fail to reject H_o
   $H_A : \mu < 10$ minutes $p-$value $= 0.0367$ b) Reject H_o
7. $H_0 : \mu = 1.5$ births critical value: $-z_{0.05} = -1.645$ Fail to reject H_0
   $H_A : \mu < 1.5$ births test statistic: $z = -0.94$
   $p-$value $= 0.1736$
8. $H_0 : \mu = 40$ hours critical value: $z_{0.05} = 1.645$ Reject H_0
   $H_A : \mu > 40$ hours test statistic: $z = 1.82$
   $p-$value $= 0.0344$
9. $H_0 : \mu = 8$ minutes critical value: $-z_{0.025} = -1.96$ Fail to reject H_0
   $H_A : \mu < 8$ minutes test statistic: $z = -1.62$
10. $H_0 : \mu = 5$ hours critical values: $\pm z_{0.005} = \pm 2.58$ Fail to reject H_0
    $H_A : \mu \neq 5$ hours test statistic: $z = 1.51$
    $p-$value $= 0.1310$