

MATHEMATICS 360-255-LW

Quantitative Methods II

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XI – Estimating proportions SOLUTIONS

1. What is the percentage of Canadians who are not exposed to second hand smoke? A sample of 200 nonsmokers revealed that only 22 of them did not show traces of a chemical that appears in the blood of people exposed to secondhand smoke. Construct a 99% confidence interval for the percentage of all Canadians not exposed to second hand smoke.

Step 1 Assumptions: $n = 200 > 20$ $n\hat{p} = 22 > 5$ and $n\hat{q} = 178 > 5$

Step 2 a) Test statistic: z

b) Level of confidence: $1 - \alpha = 0.99$ or $\alpha = 0.01$

Step 3 Point estimate: $\hat{p} = \frac{r}{n} = \frac{22}{200} = 0.11$

Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.005} = 2.58$

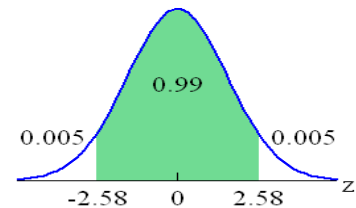
$$b) E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.58 \sqrt{\frac{22 \cdot 178}{200 \cdot 200}} = 0.0571$$

$$c) \bar{x} - E < p < \bar{x} + E$$

$$0.11 - 0.0571 < p < 0.11 + 0.0571$$

$$0.0529 < p < 0.1671$$

Step 5 The 99% confidence interval for the percentage of all Canadians not exposed to second hand smoke is 5.29% to 16.71%



2. A random sample of 250 registered voters revealed that 63 of them feel that education is the most important issue when deciding on a candidate. Construct a 90% confidence interval for the proportion of all registered voters who feel that education is the most important issue when deciding on a candidate.

Step 1 Assumptions: $n = 250 > 20$ $n\hat{p} = 63 > 5$ and $n\hat{q} = 187 > 5$

Step 2 a) Test statistic: z

b) Level of confidence: $1 - \alpha = 0.90$ or $\alpha = 0.10$

Step 3 Point estimate: $\hat{p} = \frac{r}{n} = \frac{63}{250} = 0.252$

Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.05} = 1.645$

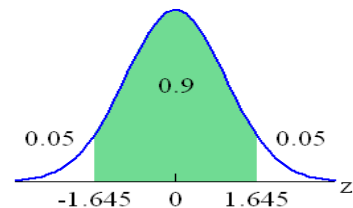
$$b) E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.645 \sqrt{\frac{63 \cdot 187}{250 \cdot 250}} = 0.0452$$

$$c) \bar{x} - E < p < \bar{x} + E$$

$$0.252 - 0.0452 < p < 0.252 + 0.0452$$

$$0.2068 < p < 0.2972$$

Step 5 The 90% confidence interval for the proportion of all registered voters who feel that education is the most important issue when deciding on a candidate is 20.68% to 29.72%



3. A local buffet restaurant has many patrons that are senior citizens. The manager of the restaurant claims that at least 60% of her customers are senior citizens. A random sample of 50 diners revealed that 33 of them are senior citizens.

- a) Construct a 95% confidence interval for the true proportion of customers that are senior citizens.

Step 1 Assumptions: $n = 50 > 20$ $n\hat{p} = 33 > 5$ and $n\hat{q} = 17 > 5$

Step 2 a) Test statistic: z

b) Level of confidence: $1 - \alpha = 0.95$ or $\alpha = 0.05$

Step 3 Point estimate: $\hat{p} = \frac{r}{n} = \frac{33}{50} = 0.66$

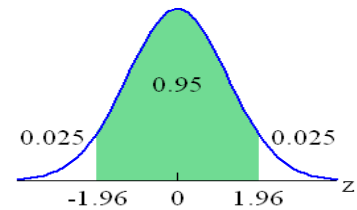
Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$

b) $E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96 \sqrt{\frac{33 \cdot 17}{50 \cdot 50}} = 0.1313$

c) $\bar{x} - E < p < \bar{x} + E$

$$0.66 - 0.1313 < p < 0.66 + 0.1313$$

$$0.5287 < p < 0.7313$$



Step 5 The 95% confidence interval for the proportion of customers who are senior citizens is 52.87% to 73.13%

- b) Is the manager's claim consistent with your interval? Explain.

Yes since it falls within the confidence interval.

4. A sample of 75 renters in Quebec City showed that 40 were under that age of 35. Construct a 99% confidence interval for the percentage of all renters that are under the age of 35.

Step 1 Assumptions: $n = 75 > 20$ $n\hat{p} = 40 > 5$ and $n\hat{q} = 35 > 5$

Step 2 a) Test statistic: z

b) Level of confidence: $1 - \alpha = 0.99$ or $\alpha = 0.01$

Step 3 Point estimate: $\hat{p} = \frac{r}{n} = \frac{40}{75} = 0.5333$

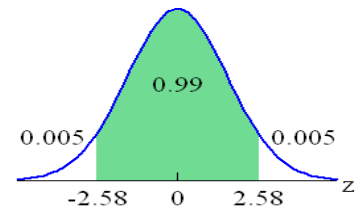
Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.005} = 2.58$

b) $E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.58 \sqrt{\frac{40 \cdot 35}{75 \cdot 75}} = 0.1486$

c) $\bar{x} - E < p < \bar{x} + E$

$$0.5333 - 0.1486 < p < 0.5333 + 0.1486$$

$$0.3847 < p < 0.6820$$



Step 5 The 99% confidence interval for the percentage of all renters that are under the age of 35 is 38.5% to 68.2%

5. A mail-order company promises its customers that the products ordered will be mailed within 72 hours after an order is placed. The quality control department at the company checks from time to time to see if this promise is fulfilled. Recently the quality department took a sample of 50 orders and found that 42 of them were mailed within 72 hours of the placement of the orders.

- a) Construct a 98% confidence interval for the percentage of all orders that are mailed within 72 hours of their placement.

Step 1 Assumptions: $n = 50 > 20$ $n\hat{p} = 42 > 5$ and $n\hat{q} = 8 > 5$

Step 2 a) Test statistic: z

b) Level of confidence: $1 - \alpha = 0.98$ or $\alpha = 0.02$

Step 3 Point estimate: $\hat{p} = \frac{t}{n} = \frac{42}{50} = 0.84$

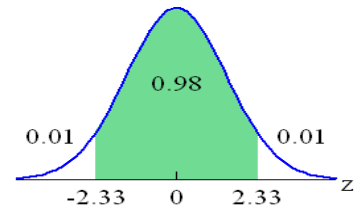
Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.01} = 2.33$

b) $E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.33 \sqrt{\frac{42 \cdot 8}{50 \cdot 50}} = 0.1208$

c) $\bar{x} - E < p < \bar{x} + E$

$$0.84 - 0.1208 < p < 0.84 + 0.1208$$

$$0.7192 < p < 0.9608$$



Step 5 The 98% confidence interval for the percentage of all orders that are mailed within 72 hours of their placement is 71.92% to 96.08%.

- b) Suppose that the confidence interval obtained in part (a) is too wide. How can the width of this interval be reduced?

Increase the sample size or reduce the level of confidence.

6. A recent poll showed that 92% of Quebec male drivers rated their driving as excellent or good. Suppose that this percentage was based on a random sample of 400 Quebec male drivers.

- a) What is the point estimate of the corresponding population proportion? What is the margin of error associated with this estimate?

Point estimate: 0.92

Margin of error: $E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} = z_{0.025} \sqrt{\frac{0.92 \cdot 0.08}{400}} = 0.0266 = 2.66\%$

- b) Find a 95% confidence interval for the corresponding proportion.

Step 1 Assumptions: $n = 400 > 20$ $n\hat{p} = 368 > 5$ and $n\hat{q} = 32 > 5$

Step 2 a) Test statistic: z

b) Level of confidence: $1 - \alpha = 0.95$ or $\alpha = 0.05$

Step 3 Point estimate: $\hat{p} = 0.92$

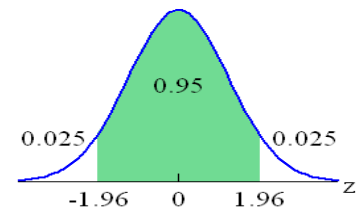
Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$

b) $E = 0.0266$

c) $\bar{x} - E < p < \bar{x} + E$

$$0.92 - 0.0266 < p < 0.92 + 0.0266$$

$$0.8934 < p < 0.9466$$



Step 5 The 95% confidence interval for the proportion of Quebec male drivers who rate their driving as excellent is 89.34% to 94.66%

7. A sample of 340 Americans were asked whether they had ever seen a UFO, and 20 said that they had. Construct a 90% confidence interval for the proportion of all Americans who have seen a UFO.

Step 1 Assumptions: $n = 340 > 20$ $n\hat{p} = 20 > 5$ and $n\hat{q} = 320 > 5$

Step 2 a) Test statistic: z

b) Level of confidence: $1 - \alpha = 0.90$ or $\alpha = 0.10$

Step 3 Point estimate: $\hat{p} = \frac{r}{n} = \frac{20}{340} = 0.059$

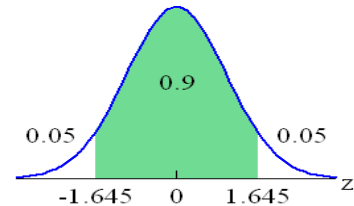
Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.05} = 1.645$

b) $E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.645 \sqrt{\frac{20 \cdot 320}{400 \cdot 400}} = 0.0210$

c) $\bar{x} - E < p < \bar{x} + E$

$$0.059 - 0.0210 < p < 0.059 + 0.0210$$

$$0.0378 < p < 0.0798$$



Step 5 The 90% confidence interval for the proportion of all Americans who have see a UFO is 3.78% to 7.98%

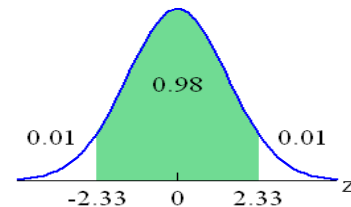
8. Tony's pizza guarantees all pizza deliveries within 30 minutes of the placement of the orders. An agency wants to estimate the proportion of all pizzas delivered within 30 minutes by Tony's. What is the most conservative estimate of the sample size that would limit the maximum error to within 2% of the population proportion for a 99% confidence interval?

$$1 - \alpha = 0.99 \quad \alpha = 0.01$$

$$z_{\frac{\alpha}{2}} = z_{0.005} = 2.58$$

$$n = \frac{\left(z_{\frac{\alpha}{2}}\right)^2}{4E^2} = \frac{2.58^2}{4(0.02)^2} = 1691.3$$

Thus 1692 deliveries.



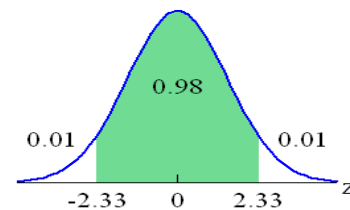
9. A consumer agency wants to estimate the proportion of all drivers who wear seat belts while driving. Assume that a preliminary study has shown that 76% of drivers wear seat belts while driving. How large should the sample size be so that the 99% confidence interval for the population proportion has a maximum error of 3%?

$$1 - \alpha = 0.99 \quad \alpha = 0.01$$

$$z_{\frac{\alpha}{2}} = z_{0.005} = 2.58$$

$$n = \frac{\left(z_{\frac{\alpha}{2}}\right)^2 p^* q^*}{E^2} = \frac{2.58^2 \cdot 0.76 \cdot 0.24}{0.03^2} = 1349.03$$

Thus 1350 deliveries.



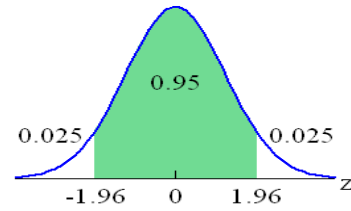
10. A researcher wants to determine the proportion of all high school students who have Internet access at home. He has no idea what the sample proportion will be. How large a sample is required to be 95% sure that the sample proportion is off by no more than 5%?

$$1 - \alpha = 0.95 \quad \alpha = 0.05$$

$$z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$$

$$n = \frac{\left(z_{\frac{\alpha}{2}}\right)^2}{4E^2} = \frac{1.96^2}{4(0.03)^2} = 384.16$$

Thus 385 high school students.



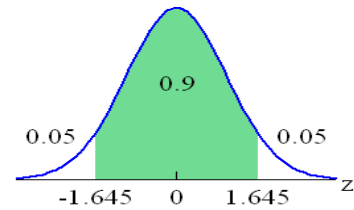
11. A student what to determine what percentage of college students smoke. How large a sample should she take to be 90% confident that her sample proportion is off by no more than 4.5%?

$$1 - \alpha = 0.90 \quad \alpha = 0.10$$

$$z_{\frac{\alpha}{2}} = z_{0.05} = 1.645$$

$$n = \frac{\left(z_{\frac{\alpha}{2}}\right)^2}{4E^2} = \frac{1.645^2}{4(0.045)^2} = 334.08$$

Thus 335 high school students



12. Consider the following news article.

More Canadians blaming U.S. policies for Sept. 11 attacks

Janice Tibbetts, CanWest News Service

Published: Monday, September 11, 2006

OTTAWA - Just over half of Canadians blame American foreign policy for the Sept. 11, 2001 terrorist attacks, showing a hardening of opinions since the one-year anniversary of the disaster, when people in this country were less inclined to attribute the bombings to U.S. meddling in certain parts of the world.

A poll conducted for Canwest News Service indicates that 53 per cent of Canadians believe the attacks were "a very specific violent reaction to foreign policies of the U.S. government."

⋮

All questions in the poll, with the exception of the one dealing with U.S. blame, were asked of 1,000 adults on Aug. 29 to 31. The results have a margin of error of plus or minus 3.1 percentage points.

- What is the point estimate for the percentage of Canadians who blame U.S. policies for Sept. 11 attacks? **53%**
- What is the size of the sample in the survey? **1000**
- What is the level of confidence in the survey? **95%**
- What is the margin of error? **3.1%**
- What is the confidence interval for the percentage of Canadians who blame U.S. policies for Sept. 11 attacks? **49.9% to 56.1%**