MATHEMATICS 360-255-LW
Quantitative Methods II
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VII – The Normal Distribution

1. Find the area under the normal curve
   a) between $z = 0$ and $z = 1.90$
   b) between $z = -1.75$ and $z = 0$
   c) between $z = 1.25$ and $z = 2.37$
   d) between $z = -2.78$ and $z = -1.53$
   e) between $z = -1.67$ and $z = 2.34$

2. Find the area under the standard normal curve
   a) to the right of $z = 1.56$
   b) to the left of $z = -1.97$
   c) to the right of $z = -2.05$
   d) to the left of $z = 1.86$

3. Find the following probabilities
   a) $P(z < 1.25)$ b) $P(z < -2.16)$ c) $P(z \le 1.07)$
   d) $P(z > 2.33)$ e) $P(z \ge -0.43)$ f) $P(z > 0.09)$
   g) $P(1.29 < z < 2.13)$ h) $P(-2.05 < z < -1.13)$ i) $P(-2 < z < 2)$
   j) $P(z = 1.65)$

4. Find the z-score(s)
   a) associated with P_{23}
   b) associated with P_{80}
   c) that bound the middle 98% of the normal distribution
   d) that bound the middle 99% of the normal distribution
   e) such that the area to the right of its value is 0.15
   f) such that the area to the left of its value is 0.30
   g) such that the area to the left of its value is 0.55

5. Find the following areas under a normal distribution curve with $\mu = 20$ and $\sigma = 4$.
   a) Area between $x = 20$ and $x = 27$
   b) Area between $x = 23$ and $x = 25$
   c) Area between $x = 9.5$ and $x = 17$

6. Determine the area of the normal distribution curve with $\mu = 55$ and $\sigma = 7$.
   a) to the right of $x = 58$
   b) to the right of $x = 43$
   c) to the left of $x = 67$
   d) to the left of $x = 24$

7. Let x be a continuous random variable that follows a normal distribution with a mean of 200 and a standard deviation of 25.
   a) Find the value of x so that the area under the normal curve to the left of x is approximately 0.6330.
   b) Find the value of x so that the area under the normal curve to the right of x is approximately 0.05.
   c) Find the value of x so that the area under the normal curve to the right of x is approximately 0.8051.
   d) Find the value of x so that the area under the normal curve to the left of x is approximately 0.015.
   e) Find the value of x so that the area under the normal curve between μ and x is approximately 0.4525 and the value of x is smaller than μ.
   f) Find the value of x so that the area under the normal curve between μ and x is approximately 0.48 and the value of x is greater than μ.

8. Suppose that the time taken to run a road race is normally distributed with a mean of 195 minutes and a standard deviation of 21 minutes. If a runner is selected at random, what is the probability that this runner will complete the this road race
   a) in less than 150 minutes
   b) in 205 to 245 minutes

9. According to Statistics Canada, Canadians aged 26 to 44 spend an average of 22 minutes traveling to work. Assume that such travel times for workers aged 26 to 44 are normally distributed with a mean of 22 minutes and a standard deviation of 5 minutes.
   a) Find the probability that a randomly selected 26-to-44 year old Canadian worker spent at least 30 minutes traveling to work?
   b) What percentage of such workers spent between 10 and 18 minutes traveling to work?

10. Reaction time is normally distributed with a mean of 0.7 seconds and a standard deviation of 0.1 seconds. Find the probability that an individual selected at random has a reaction time
    a) Greater than 0.9 seconds.
    b) Less than 0.6 seconds.
    c) Between 0.6 and 0.9 seconds.

11. The IQ of students at the Wilson Elementary School were measured recently and found to be normally distributed with a mean of 100 and a standard deviation of 15. What is the probability that a student selected at random will have an IQ
    a) Of 140 or higher?
    b) Of 120 or higher?
    c) Between 100 and 120?
    d) Of 90 or less?

12. Final averages are typically normally distributed with a mean a mean of 72 and a standard deviation of 12.5. Your professor says that the top 8% of the class will receive an A; the next 20%, a B; the next 42%, a C; the next 18%, a D; and the bottom 12%, an F.
    a) What average must you exceed to obtain an A?
    b) What average must you exceed to receive a grade better than a C?
    c) What average must you obtain to pass the course? (You'll need a D or better.)

13. Quick Start Company makes 12-volt car batteries. After many years of product testing, the company knows that the average life of a Quick Start battery is normally distributed, with a mean of 45 months and a standard deviation of 8 months.
    a) If Quick Start guarantees a full refund on any battery that fails within the 36-month period after purchase, what percentage of its batteries will the company expect to replace?
    b) If Quick Start does not want to replace more than 10% of its batteries under the full-refund guarantee policy, for how long should the company guarantee the batteries (to the nearest month)?

14. The yearly salaries of social workers are normally distributed with a standard deviation of $7215 per year. Find the mean salary of social workers if only 3% of social workers make below $25 000 a year.

15. The average annual salary of a psychologist is $56 362. If we assume that the annual salaries are normally distributed with a standard deviation of $9 506, find the following
    a) What percentage earn below $50000?
    b) What percentage earn above $75000?

16. Express Courier Service has fond that the delivery times for packages are normally distributed with mean 14 hours and standard deviation 2 hours.
    a) For a package selected at random, what is the probability that it will be delivered in 18 hours or less?
    b) What should be the guaranteed delivery time on all packages in order to be 95% sure that a given package will be delivered within this time?

17. A coin is tossed 500 times. Find the probability
    a) At least 270 tosses are heads
    b) 225 or fewer tosses are heads
    c) Between 230 tosses and 245 tosses, inclusive, are heads
    d) At least 255 tosses are heads

18. At a certain college, 60% of the students are female. Three hundred students are randomly selected to be included in the survey. Find the probability that
    a) At least 195 of the students are female
    b) 155 or fewer students are female
    c) 150 or more students are male.

19. In a recent study of high school students, about 20% of them said that they do not try as hard as they could in the classroom because they fear the disapproval of their peers. Assuming that 20% of Canadian high school students feel this way, find the probability that in a random sample of 100 Canadian high school students, 12 to 18 inclusively feel this way.

20. A fast food chain store conducted a taste survey before marketing a new hamburger. The results of the survey showed that 70% of the people who tried this hamburger liked it. Encouraged by this result, the company decided to market the new hamburger. Assume that 70% of all people like this hamburger. On a certain day, 100 customers bought this hamburger.
    a) Find the probability that exactly 65 of the 100 customers will like this hamburger.
    b) What is the probability that 60 or less of the 100 customers will like this hamburger?
    c) What is the probability that between 75 and 80 of the 100 customers will like this hamburger?

21. Stress on the job is a major concern of a large number of people who go into managerial positions. It is estimated that 80% of managers of all companies suffer from job-related stress.
    a) What is the probability that in a sample of 200 managers of companies, exactly 150 suffer from job-related stress?
    b) What is the probability that in a sample of 200 managers of companies, at least 170 suffer from job-related stress?
    c) What is the probability that in a sample of 200 managers of companies, 165 or less suffer from job-related stress?
    d) What is the probability that in a sample of 200 managers of companies, between 164 and 172 inclusively suffer from job-related stress?

ANSWERS

1. a) 0.4713 b) 0.4599 c) 0.0967 d) 0.0603 e) 0.9429
2. a) 0.0594 b) 0.0244 c) 0.9798 d) 0.9686
3. a) 0.8944 b) 0.0154 c) 0.8577 d) 0.0099
   e) 0.6664 f) 0.4641 g) 0.0819 h) 0.1090
   i) 0.9544 j) 0
4. a) –0.74 b) 0.84 c) ±2.33 d) ±2.58
   e) 1.04 f) –0.52 g) 0.13
5. a) 0.4599 b) 0.1210 c) 0.2223
6. a) 0.3336 b) 0.9564 c) 0.9564 d) 0.000
7. a) 208.50 b) 241.25 c) 178.50 d) 145.75
   e) 158.25 f) 251.25
8. a) 0.0162 b) 0.3069
9. a) 0.0548 b) 20.37%
10. a) 0.0227 b) 0.1587 c) 0.8186
11. a) 0.0038 b) 0.0918 c) 0.4082 d) 0.2514
12. a) 89.6 b) 79.2 c) 57.3
13. a) 13% b) 35 months
14. $38 564 per year
15. a) 25.14% b) 2.50%
16. a) 0.9772 b) 17.3 hours
17. a) 0.0409 b) 0.0143 c) 0.3110 d) 0.3446
18. a) 0.0436 b) 0.0019 c) 0.9998
19. 0.3354
20. a) 0.0484 b) 0.0192 c) 0.0959
21. a) 0.0151 b) 0.4065 c) 0.8340 d) 0.2540