MATHEMATICS 360-255-LW
Quantitative Methods II
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Test #3
SOLUTIONS

Answer all questions and show all your work.

Question 1 (12 points)

To measure the effectiveness of a new drug on headaches, a random sample of 5 adults were asked to rate their headache (on a scale of 1 to 10) before and after trying the new drug.

Before	8	4	9	6	7
After	3	5	6	4	5
$d = B - A$	5	-1	3	2	2

Can you conclude, at the 5% level of significance, that the new drug has any effect on headaches? Use the *p*-value approach. Assume that the ratings of headaches are normally distributed.

Step 1 Assumptions: The sampled population are normally distributed

Step 2 $H_0: \mu_d = 0$

$H_A: \mu_d \neq 0$

Step 3 a) Test statistic: t with $df = n - 1 = 4$

b) Two tailed test with $\alpha = 0.05$

Step 4 a) $t = \dfrac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \dfrac{2.2 - 0}{\frac{2.168}{\sqrt{5}}} = 2.27$

b) $2 \cdot 0.041 < p - \text{value} < 2 \cdot 0.046$

$0.082 < p - \text{value} < 0.092$

Step 5 a) $p\text{-value} > 0.082 > \alpha = 0.05$

b) Fail to reject H_0.

$\therefore$ There is not sufficient evidence at the 5% level of significance to conclude that the drug has any effect on headaches.

Question 2 (12 points)

A psychologist wishes to know if meditation helps to reduce stress among students. Two random samples were taken, the first one with 13 students who practice meditation on a regular basis, and the second one with 14 students who never practice meditation. The stress level (on a scale of 1 to 20) was then measured, and yielded a mean of 8.5 with a standard deviation of 2.3 for the group who practices meditation, and a mean of 12.1 with a standard deviation of 3.5 for the group who never practices meditation. Can you conclude, at the 5% level of significance, that meditation helps to reduce the stress level? Use the *p*-value approach. Assume that stress levels are normally distributed.

Step 1 Assumptions: The samples are independently,
Populations are normally distributed.
Equal variances

Step 2 $H_o: \mu_{nm} - \mu_m = 0$

$H_a: \mu_{nm} - \mu_m > 0$

Step 3 a) Test statistic: t with $df = 25$

b) Right-tailed test with $\alpha = 0.05$

Step 4

$$s_p = \sqrt{\frac{(n_{nm}-1)s_{nm}^2 + (n_m - 1)s_m^2}{n_{nm}+n_m-2}} = \sqrt{\frac{12 \cdot 2.3^2 + 13 \cdot 3.5^2}{25}} = 2.985$$

$$t = \frac{(\overline{x}_{mn} - \overline{x}_m) - (\mu_{mn} - \mu_m)}{s_p\sqrt{\frac{1}{n_{mn}} + \frac{1}{n_m}}} = \frac{12.1 - 8.5}{2.985\sqrt{\frac{1}{13}+\frac{1}{14}}} = 3.13$$

p-value = 0.002

Step 5 a) *p*-value = 0.002 < α = 0.05

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that meditation helps to reduce the stress level.

Question 3 (12 points)

In a survey of 700 women and 800 men, 250 women and 50 men have watched Oprah during the past month. Construct a 95% confidence interval for the difference in the proportion of men and women who have watched Oprah during the past month.

Step 1 Assumptions: $n_W = 700 > 20$ $\quad n_W \hat{p}_W = 250 > 5$ $\quad n_W \hat{q}_W = 450 > 5$

$n_M = 800 > 20$ $\quad n_M \hat{p}_M = 50 > 5$ $\quad n_M \hat{q}_M = 750 > 5$

Samples are independent.

Step 2 a) Test statistic: z

b) $1-\alpha = 0.95$ $\quad \alpha = 0.05$

Step 3 Point estimate: $\hat{p}_w - \hat{p}_m = \frac{250}{700} - \frac{50}{800} = 0.2946$

Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$

b) $E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_w \hat{q}_w}{n_w} + \frac{\hat{p}_m \hat{q}_m}{n_m}} = 1.96 \sqrt{\frac{\frac{250}{700}\frac{450}{700}}{700} + \frac{\frac{50}{800}\frac{750}{800}}{800}} = 0.0393$

c) $(\hat{p}_w - \hat{p}_m) - E < p_w - p_m < (\hat{p}_w - \hat{p}_m) - E$

$0.2946 - 0.0393 < p_w - p_m < 0.2946 + 0.0393$

$0.2554 < p_w - p_m < 0.3339$

Step 5 $\therefore$ The 95% confidence interval for the difference in the proportion of men and women who have watched Oprah during the past month is 25.5% to 33.4%.

Question 4 (12 points)

A random sample of CEGEP and university students was selected where each was asked how they get to school in the morning. Can you conclude, at the 5% level of significance, that how a student gets to school in the morning is independent of the school level a student goes to? Use the classical approach.

	University	CEGEP	
Bus	80 (75)	45 (50)	125
Car	15 (12)	5 (8)	20
Walk	25 (33)	30 (22)	55
	120	80	200

Step 1 Assumptions: The classes are all inclusive and mutually exclusive

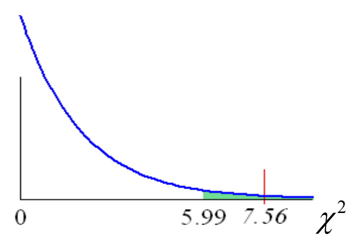
Step 2 H_o: How a student gets to school in the morning is independent of the school level.

H_A: How a student gets to school in the morning is dependent of the school level.

Step 3 a) Test statistic: χ^2 with $df = (2)(1) = 2$

b) Right-tailed test with $\alpha = 0.05$

c) $\chi^2_{(df,\alpha)} = \chi^2_{(2,0.05)} = 5.99$



Step 4

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(80-75)^2}{75} + \frac{(45-50)^2}{50} + \frac{(15-12)^2}{12} + \frac{(5-8)^2}{8} + \frac{(25-33)^2}{33} + \frac{(30-22)^2}{22}$$

$$= 7.56$$

Step 5 a) χ^2 is in the critical region

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that how a student gets to school in the morning is dependent of the school level.

Question 5 (12 points)

The owner of a psychotherapy clinic is studying the sometimes large spread in waiting time for patients to obtain an appointment for consultation. In a random sample of 25 patients, the standard deviation for the waiting times was 3.4 days. Assuming that the waiting times are normally distributed, find a 95% confidence interval for the variance and for the standard deviation of the waiting time for patients to obtain an appointment for consultation.

Step 1 Assumptions: The population is normally distributed

Step 2 a) Test statistic: χ^2 with $df = 25 - 1 = 24$

b) Level of confidence: $1-\alpha = 0.95$ or $\alpha = 0.05$

Step 3 Point Estimate: $s = 3.4$ days

$$s^2 = 3.4^2 = 11.56 \text{ days}^2$$

Step 3 a) $\chi^2_{\left(df,1-\frac{\alpha}{2}\right)} = \chi^2_{(24,0.975)} = 12.40$

$$\chi^2_{\left(df,\frac{\alpha}{2}\right)} = \chi^2_{(24,0.025)} = 39.36$$

b) $$\frac{(n-1)s^2}{\chi^2_{\left(df,\frac{\alpha}{2}\right)}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\left(df,1-\frac{\alpha}{2}\right)}}$$

$$\frac{24\cdot 3.4^2}{39.36} < \sigma^2 < \frac{24\cdot 3.4^2}{12.40}$$

$$7.05 < \sigma^2 < 22.37$$

$$2.65 < \sigma < 4.73$$

Step 4 The 95% confidence interval for the variance of waiting times is 7.05 days^2 to 22.37 days^2, and the standard deviation is 2.65 days to 4.73 days.

Formulas

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

$$E = t_{\left(df, \frac{\alpha}{2}\right)} \frac{s_d}{\sqrt{n}}$$

$$z = \frac{\left(\bar{x}_1 - \bar{x}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$E = z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$t = \frac{\left(\bar{x}_1 - \bar{x}_2\right) - \left(\mu_1 - \mu_2\right)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$E = t_{\left(df, \frac{\alpha}{2}\right)} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$s_p = \sqrt{\frac{\left(n_1 - 1\right) s_1^2 + \left(n_2 - 1\right) s_2^2}{n_1 + n_2 - 2}}$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_p \hat{q}_p \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

$$\chi^2 = \sum \frac{\left(O - E\right)^2}{E}$$

$$\chi^2 = \frac{\left(n - 1\right) s^2}{\sigma^2} \quad \text{with } df = n - 1$$

$$\frac{\left(n - 1\right) s^2}{\chi^2_{\left(df, \frac{\alpha}{2}\right)}} < \sigma^2 < \frac{\left(n - 1\right) s^2}{\chi^2_{\left(df, 1 - \frac{\alpha}{2}\right)}}$$