MATHEMATICS 360-255-LW
Quantitative Methods II
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Friday October 30, 2009

Test #2
SOLUTIONS

Answer all questions and show all your work.

Question 1 (12 points)

Serge, a caffeine addict, drinks, on average, 9 coffees per day with a standard deviation of 3.2 coffees. Assuming that the number of coffees Serge drinks in a day is normally distributed,

a) what is the probability that on a given day there will be between 8 and 12 coffees?

$$P(8<x<12)=P(-0.31<z<0.94) = 0.8264-0.3783 = 0.4481$$

$$z_1=\frac{x-\mu}{\sigma}=\frac{8-9}{3.2}=-0.31 \qquad z_2=\frac{x-\mu}{\sigma}=\frac{12-9}{3.2}=0.94$$

b) what is the probability that on a given day there will be more than 10 coffees?

$$P(x>10)=P(z>0.31) = 1-0.6217 = 0.3783$$

$$z=\frac{x-\mu}{\sigma}=\frac{10-9}{3.2}=0.31$$

c) what is the probability that on a given day there will be less that 5 coffees?

$$P(x<5)=P(z<-1.25) = 0.1056$$

$$z=\frac{x-\mu}{\sigma}=\frac{5-9}{3.2}=-1.25$$

d) 5% of the time, he drinks less than how many coffees?

Area to the left = 0.05

$z=-1.645$

$$x=\mu+z\sigma=9-1.645\cdot 3.2=3.74 \text{ coffees}$$

Question 2 (4 points)

The results for a particular test which measures anxiety are normally distributed with a mean of 200 and a standard deviation of 40 (where a higher score indicates a higher level of anxiety). If the 10% of people who scored the highest are made to see a psychologist for further testing, what is the score needed to be made to see a psychologist?

Area to the left = 0.90

$$z = 1.28$$

$$\begin{aligned} x &= \mu + z\sigma \\ &= 200 + 1.28 \cdot 40 \\ &= 251.2 \end{aligned}$$

Thus a score of 251 is needed.

Question 3 (6 points)

A recent survey revealed that 75% of SLC students have a cell phone. In a group of 150 students, what is the probability that less than 100 have a cell phone.

Assumptions: $np = 150 \cdot 0.75 = 112.5 > 5$

$nq = 150 \cdot 0.25 = 37.5 > 5$

$$\mu = np = 112.5$$

$$\sigma^2 = npq = 28.125$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{28.125} = 5.303$$

Thus $B(150, 0.75) \sim N(112.5, 28.125)$

$$\begin{aligned} P(r < 100) &= P(x < 99.5) \\ &= P(z < -2.45) \\ &= 0.0071 \end{aligned}$$

$$z = \frac{x - \mu}{\sigma} = \frac{99.5 - 112.5}{5.303} = -2.45$$

Question 4 (4 points)

It has been shown that the score a student obtains on a QM II exam is normally distributed with a mean of 75 and a standard deviation of 12. What is the probability that the average grade in a class with 15 students is higher than 80?

$$\begin{aligned} P(\overline{x} > 80) &= P(z > 1.61) \\ &= 1 - 0.9463 \\ &= 0.0537 \end{aligned}$$

$$z = \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{80 - 75}{\frac{12}{\sqrt{15}}} = 1.61$$

Question 5 (10 points)

Serge wants to estimate the number of coffees students at SLC drink per day. For this, he takes a random sample of SLC. Here are the results.

0	1	1	1	2	2	2	3	4

Find a 95% confidence interval for the mean number of coffees SLC students drink per day, assuming the number of coffees students drink is normally distributed.

Step 1 Assumptions: Population is normally distributed

Step 2 a) Test statistic: t with $df = 9 - 1 = 8$

b) Level of confidence: $1 - \alpha = 0.95$ or $\alpha = 0.05$

Step 3 Point estimate: $\bar{x} = 1.778$ coffees

Step 4 a) $t_{\left(df, \frac{\alpha}{2}\right)} = t_{(8, 0.025)} = 2.306$

b) $E = t_{\left(df, \frac{\alpha}{2}\right)} \frac{s}{\sqrt{n}} = 2.306 \frac{1.202}{\sqrt{9}} = 0.924$

a) $$\bar{x} - E < \mu < \bar{x} + E$$

$$1.779 - 0.924 < \mu < 1.779 + 0.924$$

$$0.855 < \mu < 2.703$$

Step 5 The 95% confidence interval for mean number of coffees SLC students drink per day is 0.855 to 2.703 coffees.

Question 6 (10 points)

A sociologist claims that students spend on average 2 hours (120 minutes) on the internet per day. To test this claim, a random sample of 50 students is taken, and the mean time spent on the internet per day was 104.5 minutes. Can you conclude, from the sample, that the sociologist' claim is false? Use the *p*-value approach with a 5% level of significance. Assume that the population standard deviation is 32.5 minutes.

Step 1 Assumptions: $n = 50 \geq 30$

Step 2 $H_o: \ \mu = 120$ min per day

$H_a: \ \mu \neq 120$ min per day

Step 3 a) Test statistic: z

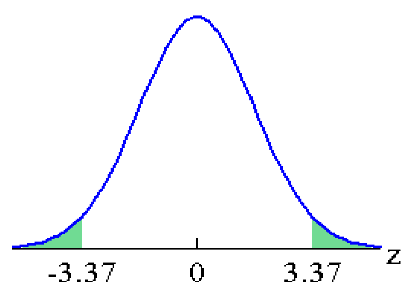
b) Two-tailed test with $\alpha = 0.05$

Step 4 a) $z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{104.5 - 120}{\frac{32.5}{\sqrt{50}}} = -3.37$

b) p-value = $= 2P(z < -3.37)$

$= 2(0.0004)$

$= 0.0008$



Step 5 a) p-value $= 0.0008 < \alpha = 0.05$

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that students do not spend on average 120 minutes on the internet per day.

Question 7 (10 points)

A sociologist claims that more that 40% of CEGEG students have an IPod. A random sample of 250 students was taken, where 115 have an IPod. Can you conclude, at the 1% level of significance, that more than 40% of CEGEP students have an IPod? Use the classical approach.

Step 1 Assumptions: $n = 250 > 20$

$np = 250 \cdot 0.4 = 100 > 5$

$nq = 250 \cdot 0.6 = 150 > 5$

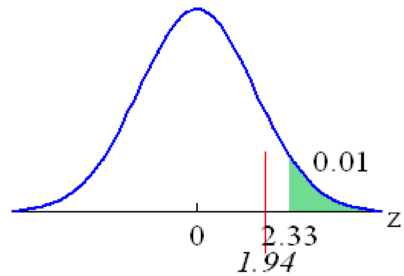
Step 2 $H_o: \ p = 0.4$

$H_A: \ p > 0.4$

Step 3 a) Test statistic: z

b) Right-tailed test with $\alpha = 0.01$

c) $z_\alpha = z_{0.01} = 2.33$



Step 4 a) $z = \dfrac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \dfrac{\frac{115}{250} - 0.4}{\sqrt{\frac{0.4 \cdot 0.6}{250}}} = 1.94$

Step 5 a) z is not in the critical region

b) Fail to reject H_o.

$\therefore$ There is not sufficient evidence at the 1% level of significance to conclude that more than 40% of CEGEP students have an IPod.

Question 8 (4 points)

The Liberal Party of Canada wants to estimate the proportion of Canadians who will vote for them at the next federal election. How large should a sample be taken so that the estimate, at a 95% level of confidence, is within 2.5% of the true proportion?

$z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$

$n = \dfrac{z_{\frac{\alpha}{2}}^2}{4E^2} = \dfrac{1.96^2}{4 \cdot 0.025^2} \simeq 1537$

Hence the sample should have 1537 Canadians.