MATHEMATICS 360-255-LW
Quantitative Methods II
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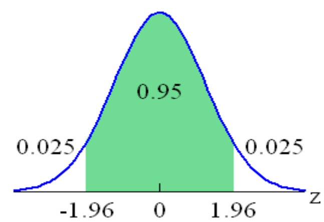
Fall 2009

IX – Estimating μ (known σ)
SOLUTIONS

1. A random sample of 150 college students was taken, where each was asked for the number of hours per week they work at a paid job outside of school. A mean of 12.4 hours worked per week was obtained. Construct a 95% confidence interval for the mean number of hours worked per week by college students, assuming that the population standard deviation is 7.3 hours.

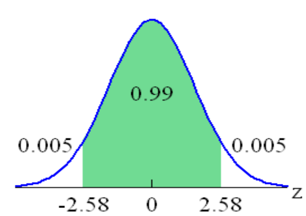
Step 1 Assumptions: $n = 150 \geq 30$
Step 2 a) Test statistic: z
b) Level of confidence: $1-\alpha = 0.95$ or $\alpha = 0.05$
Step 3 Point estimate: $\overline{x} = 25.4$ hours
Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$
b) $E = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 1.96 \frac{7.3}{\sqrt{150}} = 1.17$
c) $\overline{x} - E < \mu < \overline{x} + E$
$12.41 - 1.17 < \mu < 12.41 + 1.17$
$11.24 < \mu < 13.58$



Step 5 The 95% confidence interval for the mean number of hours worked per week by college students is 11.24 to 13.58 hours.

2. Twenty randomly selected college students were asked how many cavities they had. A mean of 3.21 cavities was obtained. Construct a 99% confidence interval for the mean number of cavities for all college students, assuming that the number of cavities is normally distributed, and that $\sigma = 1.65$.

Step 1 Assumptions: Population normally distributed
Step 2 a) Test statistic: z
b) Level of confidence: $1-\alpha = 0.99$ or $\alpha = 0.01$
Step 3 Point estimate: $\overline{x} = 3.21$ cavities
Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.005} = 2.58$
b) $E = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 2.58 \frac{1.65}{\sqrt{20}} = 0.95$
c) $\overline{x} - E < \mu < \overline{x} + E$
$3.21 - 0.95 < \mu < 3.21 + 0.95$
$2.26 < \mu < 4.16$



Step 5 The 99% confidence interval for the mean number of cavities of all college students is 2.26 to 4.16 cavities.

3. How large are math classes in CEGEP's? A random sample of 100 math classes had a mean of 28.3 students. Assuming that $\sigma = 4.5$ students, construct a 92% confidence interval for the mean class size for all CEGEP math classes.

Step 1 Assumptions: $n = 100 \geq 30$

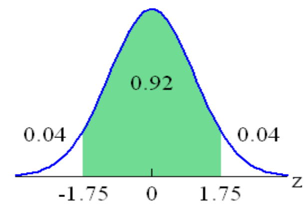
Step 2 a) Test statistic: z

b) Level of confidence: $1-\alpha = 0.92$ or $\alpha = 0.08$

Step 3 Point estimate: $\overline{x} = 28.3$ students

Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.04} = 1.75$

b) $E = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 1.75 \frac{4.5}{\sqrt{100}} = 0.79$



c) $\overline{x} - E < \mu < \overline{x} + E$

$28.3 - 0.79 < \mu < 28.3 + 0.79$

$27.51 < \mu < 29.09$

Step 5 The 92% confidence interval for the mean class size for all CEGEP math classes is 27.51 to 29.09 students.

4. A survey of Canadians planning a summer vacation in 2009 revealed a mean planned expenditure of \$3048. Assume that this mean is based on a random sample of 300 Canadians who were planning summer vacations in 2009 Previous studies have shown that the population standard deviation is \$710. Construct a 99% confidence interval for the mean planned expenditure by all Canadians taking a summer vacation in 2009.

Step 1 Assumptions: $n = 300 \geq 30$

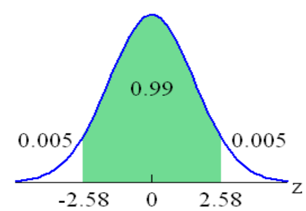
Step 2 a) Test statistic: z

b) Level of confidence: $1-\alpha = 0.99$ or $\alpha = 0.01$

Step 3 Point estimate: $\overline{x} = 3048\,\$$

Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.005} = 2.58$

b) $E = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 2.58 \frac{710}{\sqrt{300}} = 105.76$



c) $\overline{x} - E < \mu < \overline{x} + E$

$3048 - 105.76 < \mu < 3048 + 105.76$

$2942.24 < \mu < 3153.76$

Step 5 The 99% confidence interval for the mean planned expenditure by all Canadians taking a summer vacation in 2009 is \$2942.24 to \$3153.76.

5. Do people who stop smoking tend to gain weight? A study (later published in a medical journal) was undertaken to answer this question. The authors of this study collected data on a random sample of 15 men over the age of 35 who had quit smoking during the past 10 years and found that these men had gained an average of 5.28 kilograms since quitting smoking. Assuming that weight gains are normally distributed with a population standard deviation of 0.59 kilogram, construct a 98% confidence interval for the corresponding population mean.

Step 1 Assumptions: $n = 315 \geq 30$

Step 2 a) Test statistic: z

b) Level of confidence: $1-\alpha = 0.98$ or $\alpha = 0.02$

Step 3 Point estimate: $\bar{x} = 5.28$ kg

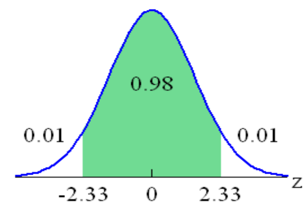
Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.01} = 2.33$

b) $E = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 2.33 \frac{0.59}{\sqrt{315}} = 0.077$

c) $\bar{x} - E < \mu < \bar{x} + E$

$5.28 - 0.077 < \mu < 5.28 + 0.077$

$5.203 < \mu < 5.357$



Step 5 The 98% confidence interval for the mean weight gain by men over the age of 35 who had quit smoking during the past 10 years is 5.203 to 5.357 kg.

6. The CAA estimated that Canadians planned to spend an average of 4.83 nights away on vacation in 2008. Suppose that this mean was based on a sample of 24 Canadians who planned vacations, that the population standard deviation was 1.52 nights, and that the number of nights is normally distributed. Construct a 97% confidence interval for the mean length of vacations Canadians planned in 2008.

Step 1 Assumptions: Population normally distributed

Step 2 a) Test statistic: z

b) Level of confidence: $1-\alpha = 0.97$ or $\alpha = 0.03$

Step 3 Point estimate: $\bar{x} = 4.8$ days

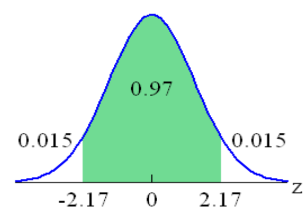
Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.015} = 2.17$

b) $E = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 2.17 \frac{1.5}{\sqrt{24}} = 0.67$

c) $\bar{x} - E < \mu < \bar{x} + E$

$4.83 - 0.67 < \mu < 4.83 + 0.67$

$4.16 < \mu < 5.50$



Step 5 The 97% confidence interval for the mean length of vacations Canadians planned in 2008 is 4.16 to 5.50 nights.

7. A random sample of 60 night school students' ages is obtained in order to estimate the mean age of night school students. If the sample mean is 25.3 years, construct a 95% confidence interval for the mean age of night school students, assuming $\sigma = 4$ years.

Step 1 Assumptions: $n = 60 \geq 30$

Step 2 c) Test statistic: z

d) Level of confidence: $1-\alpha = 0.95$ or $\alpha = 0.05$

Step 3 Point estimate: $\bar{x} = 25.3$ years old

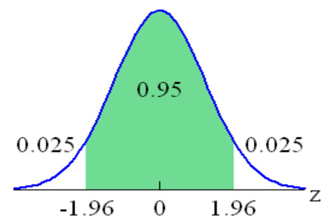
Step 4 d) $z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$

e) $E = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 1.96 \frac{4}{\sqrt{60}} = 1.01$

f) $\bar{x} - E < \mu < \bar{x} + E$

$25.3 - 1.01 < \mu < 25.3 + 1.01$

$24.29 < \mu < 26.31$



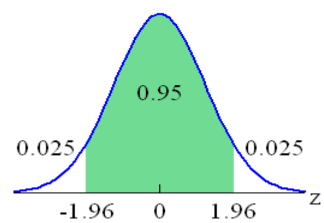
Step 5 The 95% confidence interval for the mean age of night school students is 24.29 to 26.31 years old.

8. A high-tech company wants to estimate the mean number of years of college education its employees have completed. A good estimate of the standard deviation for the mean number of years of college is 1.0. How large a sample needs to be taken to estimate μ to within one quarter of a year with 99% confidence?

$1-\alpha = 0.99 \qquad \alpha = 0.01$

$z_{\frac{\alpha}{2}} = z_{0.005} = 2.58$

$$n = \left(\frac{z_{\frac{\alpha}{2}}\sigma}{E}\right)^2 = \left(\frac{2.58 \cdot 1}{0.25}\right)^2 = 106.2$$



Thus 107 employees.

9. A researcher wants to determine a 95% confidence interval for the mean number of hours that high school students spend doing homework per week. A preliminary study showed that the standard deviation for hours spent per week by all high school students doing homework is 0.7. How large a sample should the researcher select so that the estimate will be within 0.15 hours of the population mean?

$1-\alpha = 0.95 \qquad \alpha = 0.05$

$z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$

$$n = \left(\frac{z_{\frac{\alpha}{2}}s}{E}\right)^2 = \left(\frac{1.96 \cdot 0.7}{0.15}\right)^2 = 83.7$$

Thus 84 high school students

10. A department store manager wants to estimate at a 90% confidence level the mean amount spent by all customers at this store. From an earlier study, the manager knows that the standard deviation of amounts spent by customers at this store is \$27. What sample size should he choose so that the estimate is within \$3 of the population mean?

$1-\alpha = 0.90 \qquad \alpha = 0.10$

$z_{\frac{\alpha}{2}} = z_{0.05} = 1.645$

$$n = \left(\frac{z_{\frac{\alpha}{2}}\sigma}{E}\right)^2 = \left(\frac{1.645 \cdot 27}{3}\right)^2 = 219.1$$

Thus 220 customers.