

MATHEMATICS 360-255-LW

Quantitative Methods II

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III – Rules of Probability

- Two cards are drawn from a deck of 52 cards. What is the probability that
 - both are clubs
 - both are Jacks.
- A single card is drawn from a deck of 52 cards. What is the probability that the card is
 - a king of diamonds?
 - a king or a diamond?
 - a face card?
 - a red face card?
 - not an ace?
- An urn contains 20 marbles, of which 5 are red, 6 are blue, 7 are yellow and 2 are green. If two marbles are chosen at random, what is the probability that
 - both are blue?
 - one is blue and the other is red?
 - both are of the same color?
 - the two marbles are not the same color?
- In a refugee camp in Rwanda, it was found that 90% of the refugees came to escape political oppression, 80% came to escape abject poverty, and 70% came to escape both. What is the probability that a refugee in the camp was neither poor nor seeking political asylum?
- A survey conducted about job satisfaction showed that 20% of workers are not happy with their current job. Assume that this result is true for the population of all workers. Two workers are selected at random, and it is observed whether or not they are happy with their current jobs. Find the probability that in this sample of two workers
 - both are not happy with their current jobs
 - at least one of them is happy with the current job
- Suppose a birth control pill is 99% effective in preventing pregnancy.
 - What is the probability that none of 100 women using the pill will become pregnant?
 - What is the probability that at least one woman per 100 users will become pregnant?
- A company has installed a generator to back up the power in case there is a power failure. The probability that there will be a power failure during a snowstorm is 0.30. The probability that the generator will stop working during a snowstorm is 0.09. What is the probability that during a snowstorm the company will lose both sources of power?
- An old age home employs 65 people. Eight of the 30 men and 21 of the 35 women are nurses. What is the probability that an employee picked at random is a man or is a nurse?

9. At the first meeting of a committee to plan a local Lunar New Year celebration, the persons attending are 3 Chinese men, 4 Chinese women, 3 Vietnamese women, 2 Vietnamese men, 4 Korean women, and 2 Korean men. A chairperson is selected at random. Find the probability that the chairperson is the following
- Chinese
 - Korean or a woman
 - A man or Vietnamese
 - Chinese or Vietnamese
 - Korea and a woman.
10. Two thousand random selected adults were asked if they thing they are financially better off than their parents. The following table gives the two-way classification of the responses based on the education levels of the persons included in the survey and whether they are financially better off, the same, or worse off than their parents.

	Education Level		
	High school or less	CEGEP	More than CEGEP
Better off	140	450	420
Same	60	250	110
Worse off	200	300	70

Suppose one adult is selected at random from these 2000 adults. Find the following probabilities

- P(better off or CEGEP)
 - P(More that CEGEP or worse off)
 - P(better off or worse off)
 - P(better off given CEGEP)
 - Are the events better off and CEGEP independent?
 - Are the events better off and CEGEP mutually exclusive?
11. A random sample of 80 lawyers was taken, and they were asked if they are in favor of against capital punishment. The following table gives the two-way classification of theses 80 lawyers.

	Favors Capital Punishment	Opposes Capital Punishment
Male	32	26
Female	13	9

If one lawyer is randomly selected from this group, find the probability that this lawyer

- favors capital punishment
- is a female
- opposes capital punishment given that the lawyer is a female
- is a male given that he favors capital punishment
- is a female and favors capital punishment
- opposes capital punishment or is a male
- Are the events “female” and “opposes capital punishment” independent?
- Are they mutually exclusive?

12. It is estimated that 5% of a large consignment of eggs in a certain supermarket are broken.
- What is the probability that a customer who randomly selects a dozen of these eggs receives at least one broken egg?
 - What is the probability that a customer who selects these eggs at random will have to check more than three cartons before finding a carton without any broken eggs? (Each carton contains a dozen eggs).
13. Alex, Bill and Joe each in turn toss a balanced coin. The first one to throw a head wins.
- What are their respective chances of winning if each tosses only one time?
 - What are their respective chances of winning if they continue, when there is no winner, giving a maximum of two tosses each?
14. The probability that a certain door is locked is 0.6. The key to the door is one of five unidentified keys hanging of a key rack. Two keys are randomly selected before approaching the door. What is the probability that the door may be opened without returning for another key?
15. A survey has shown that 65% of the women in a certain community work outside the home. Of these women, 54% are married, while 76% of the women who do not work outside the home are married. Find the probability that a women chosen at random is
- Married?
 - A single woman working outside the home?
16. A smooth-talking young man has a $\frac{1}{3}$ probability of talking a policeman out of giving him a speeding ticket. The probability that he is stopped for speeding during a given weekend is $\frac{1}{2}$. Find the probability that
- he will receive no speeding tickets on a given weekend?
 - he will receive no speeding tickets on 3 consecutive weekends?
17. Five black balls and four white balls are placed in an urn. Two balls are then drawn in succession. What is the probability that the second ball drawn is a white ball
- if the second ball is drawn without replacing the first?
 - If the first ball is replaced before the second is drawn?
18. Two cards are drawn in succession without replacement of a deck of 52 cards.
- What is the probability that the first card is a heart given that the second card is a heart?
 - What is the probability that the first card is a heart given that the second card is a diamond?
 - What is the probability that the first card is a jack given that the second card is an ace?

19. Urn A contains four white and six black balls. Urn B contains three white and five black balls. A ball is drawn from urn A and then transferred to urn B. A ball is then drawn from urn B.
- What is the probability that the transferred ball was white given that the second ball drawn was white?
 - What is the probability that the transferred ball was black given that the second ball drawn was white?
20. As accounts manager in your company, you classify 75% of your customers as "good credit" and the rest as "risky credit" depending on their credit rating. Customers in the "risky" category allow their accounts to go overdue 55% of the time on average, whereas those in the "good" category allow their accounts to become overdue only 10% of the time. What percentage of overdue accounts is held by customers in the "risky credit" category?
21. A witness sees a crime involving a taxi in Cabcity. The witness says that the taxi is green. It is known from previous research that witnesses are correct 80% of the time when making such statements. The police also know that 75% of the taxis in Cabcity are green, the other 25% being blue. What is the probability that a green taxi was involved in the crime?
22. A study conducted by a Housing Agency in Montreal revealed the accompanying information concerning the age distribution of renters within the city.

Age	Percentage of adult population	Percentage of group who are renters
18-44	0.51	0.58
45-64	0.31	0.45
65 and over	0.18	0.60

- What is the probability that an adult selected at random from this population is a renter?
- If a renter is selected at random, what is the probability that he or she is in the 18-44 age bracket?
- If a renter is selected at random, what is the probability that he or she is 45 years of age or older?

ANSWERS

1. a) $\frac{1}{17}$ b) $\frac{1}{221}$
2. a) $\frac{1}{52}$ b) $\frac{4}{13}$ c) $\frac{3}{13}$ d) $\frac{3}{26}$ e) $\frac{12}{13}$
3. a) $\frac{3}{38}$ b) $\frac{3}{19}$ c) $\frac{47}{190}$ d) $\frac{143}{190}$
4. 0
5. a) 0.04 b) 0.96
6. a) 0.366 b) 0.634
7. 0.027
8. $\frac{51}{65}$
9. a) $\frac{7}{18}$ b) $\frac{13}{18}$ c) $\frac{5}{9}$ d) $\frac{2}{3}$ e) $\frac{2}{9}$
10. a) $\frac{39}{50}$ b) $\frac{11}{20}$ c) $\frac{79}{100}$ d) $\frac{9}{20}$
 - e) No since $P(\text{better off}) = \frac{101}{200} \neq P(\text{better off given CEGEP}) = \frac{9}{20}$
 - f) No since $P(\text{better off and CEGEP}) = \frac{3}{40} \neq 0$
11. a) $\frac{9}{16}$ b) $\frac{11}{40}$ c) $\frac{9}{35}$ d) $\frac{32}{45}$ e) $\frac{13}{80}$ f) $\frac{67}{80}$
 - g) No since $P(\text{female}) = \frac{11}{40} \neq P(\text{female given opposes capital punishment}) = \frac{9}{26}$
 - h) No since $P(\text{female and opposes capital punishment}) = \frac{9}{80} \neq 0$
12. a) 0.460 b) 0.097
13. a) $A = \frac{1}{2}$ $B = \frac{1}{4}$ $J = \frac{1}{8}$ b) $A = \frac{9}{16}$ $B = \frac{9}{32}$ $C = \frac{9}{64}$
14. 0.64
15. a) 0.617 b) 0.299
16. a) $\frac{2}{3}$ b) $\frac{8}{27}$
17. a) $\frac{4}{9}$ b) $\frac{4}{9}$
18. a) $\frac{4}{17}$ b) $\frac{13}{51}$ c) $\frac{4}{51}$
19. a) $\frac{8}{17}$ b) $\frac{9}{17}$
20. 64.7%
21. 0.923
22. a) 0.543 b) 0.544 c) 0.456