MATHEMATICS 360-255-LW
Quantitative Methods II
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Fall 2009

Assignment 3
SOLUTIONS

This assignment is due on **Friday November 20, 2009** at the beginning of the class. Complete solutions are expected.

Question 1 (7 points)

A psychologist wants to determine if playing video games which contain violence increases the level of aggression in children. A random sample of nine children was taken where each was given the Aggression Assessment Test (where scores range from 0 to 40, with a higher score indicating a higher the level of aggression) before and after playing a video game containing violence for one hour. Here are the results.

Before	15	19	17	31	12	27	31	23	22
After	13	21	27	36	18	24	37	23	27
$d = A - B$	-2	2	10	5	6	-3	6	0	5

Construct a 95% interval for the mean difference in the level of aggression before and after playing a video game containing violence for one hour. Assume that levels of aggression are normally distributed.

Step 1 Assumptions: The populations are normally distributed

Step 2 a) Test statistic: t with $df = n - 1 = 8$

b) Level of confidence: $1 - \alpha = 0.95$ or $\alpha = 0.05$

Step 3 Point estimate: $\bar{d} = 3.22$

Step 4 a) $t_{\left(df, \frac{\alpha}{2}\right)} = t_{(8, 0.025)} = 2.306$

b) $E = t_{\left(df, \frac{\alpha}{2}\right)} \frac{s_d}{\sqrt{n}} = 2.306 \frac{4.266}{\sqrt{9}} = 3.279$

c) $\bar{d} - E < \mu_d < \bar{d} + E$

$3.22 - 3.28 < \mu_d < 3.22 + 3.28$

$-0.06 < \mu_d < 6.50$

Step 5 The 95% confidence interval for the mean difference in the level of aggression before and after playing a video game containing violence for one hour is -0.06 to 6.50.

Question 2 (7 points)

Sandy, a psychologist, is interested in studying the effect smoking marijuana on a regular basis has on a standard intelligence test (IQ test). A random group of 11 people who smoke marijuana on a regular basis is taken, along with a random group of 16 people who do not smoke marijuana on a regular basis. Each person is given an IQ test. Here are the results obtained.

Smoke	113	99	85	89	111	110	71	96	115	108	81					
Do not smoke	100	106	94	102	91	98	115	109	116	81	106	89	88	119	104	107

Construct a 90% confidence interval for the difference in the mean IQ score between people who smoke marijuana on a regular basis, and those who don't. Assume that the populations are normally distributed.

Step 1 Assumptions: The populations are normally distributed.
The samples are independent.
The variances are equal

Step 2 a) Test statistic: t with $df = n_{NS} + n_S - 2 = 16 + 11 - 2 = 25$

b) $1 - \alpha = 0.90$

Step 3 Point Estimate: $\bar{x}_{NS} - \bar{x}_S = 101.565 - 98 = 3.563$

Step 4

$$s_p = \sqrt{\frac{(n_{NS}-1)s_{NS}^2 + (n_S - 1)s_S^2}{n_{NS} + n_S - 2}} = \sqrt{\frac{15 \cdot 10.869^2 + 10 \cdot 14.832^2}{25}} = 12.605$$

$$t_{\left(df, \frac{\alpha}{2}\right)} = t_{(25, 0.05)} = 1.708$$

$$E = t_{\left(df, \frac{\alpha}{2}\right)} s_p \sqrt{\tfrac{1}{n_{NS}} + \tfrac{1}{N_S}} = 1.703 \cdot 12.605 \sqrt{\tfrac{1}{16} + \tfrac{1}{11}} = 8.432$$

$$\bar{x}_{NS} - \bar{x}_S - E < \mu_{NS} - \mu_S < \bar{x}_{NS} - \bar{x}_S + E$$

$$3.563 - 8.432 < \mu_{NS} - \mu_S < 3.563 + 8.432$$

$$-4.870 < \mu_{NS} - \mu_S < 11.995$$

Step 5 $\therefore$ The 90% confidence interval for the difference in the mean IQ score between people who don't smoke marijuana on a regular basis, and those who do is -4.87 to 11.99.

Question 3 (7 points)

A social psychologist investigated whether women smile more often than men. She videotaped random samples of men and women while interacting, and noted the number of smiles emitted by each sex during a 5-minute interaction. The following results were obtained

Men	8	12	10	7	2	1	15	9	8	9	12	10	8	10	11	7	3	4
	2	10	8	7	9	12	14	1	6	0	14	2	9	7	5	6	10	
Women	7	9	15	14	12	0	19	19	11	16	8	10	15	14	19	22	1	6
	9	10	13	7	19	15	10	16	8	14	17	21	22	15				

Construct a 90% confidence interval for the difference in the mean number of smiles emitted by men and women. Assume that the population standard deviation for the number of smiles for men is 3.5 and for women 5.5.

Step 1 Assumptions: $n_m = 35 \geq 30$, $n_w = 32 \geq 30$

The samples are independent.

Step 2 a) Test statistic: z

b) Level of confidence: $1-\alpha = 0.90$ or $\alpha = 0.05$

Step 3 Point estimate: $\bar{x}_w - \bar{x}_m = 12.91 - 7.66 = 5.249$ smiles

Step 4 a) $z_{0.05} = 1.645$

b) $E = z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_m^2}{n_m} + \frac{\sigma_w^2}{n_w}} = 1.645\sqrt{\frac{3.5^2}{35} + \frac{5.5^2}{32}} = 1.87$

c) $(\bar{x}_w - \bar{x}_m) - E < \mu_w - \mu_m < (\bar{x}_w - \bar{x}_m) + E$

$$5.25 - 1.87 < \mu_w - \mu_m < 5.25 + 1.87$$

$$3.38 < \mu_w - \mu_m < 7.13$$

Step 5 The 90% confidence interval for the mean difference in the difference in the mean number of smiles emitted by men and women is 3.38 to 7.13 smiles.

Question 4 (7 points)

Henry, a psychologist, hypothesizes that people who are allowed to sleep for only four hours will score significantly lower than people who are allowed to sleep for eight hours on a cognitive skills test. He brings twenty-four participants into his sleep lab and randomly assigns them to one of two groups. In one group he has participants sleep for eight hours and in the other group he has them sleep for four. The next morning he administers the HCAT test (Henry's Cognitive Ability Test) to all participants. Here are the scores obtained (scores on the HCAT range from 1-20 with high scores representing better performance).

Eight hours of sleep:	15	13	19	11	14	21	13	13	18	18	13	13
Four hours of sleep:	13	3	6	17	10	3	9	10	11	15	13	20

Test Henry's hypothesis that people who are allowed to sleep for only four hours will score significantly lower than people who are allowed to sleep for eight hours on a cognitive skills test. Use the *p*-value approach with a 5% level of significance. Assume that the populations are normally distributed.

Step 1 Assumptions: The populations are normally distributed.
The samples are independent.
The variances are equal

Step 2 $H_O: \mu_{Eight} - \mu_{Four} = 0$

$H_A: \mu_{Eight} - \mu_{Four} > 0$

Step 3 a) Test statistic: t with $df = n_{Eight} + n_{Four} - 2 = 12 + 12 - 2 = 22$

b) Right-tailed test with $\alpha = 0.05$

Step 4

$$s_p = \sqrt{\frac{(n_{Eight}-1)s_{Eight}^2 + (n_{Four}-1)s_{Four}^2}{n_{Eight}+n_{Four}-2}} = \sqrt{\frac{11\cdot 3.118^2 + 11\cdot 5.219^2}{22}} = 4.299$$

$$t = \frac{\bar{x}_{Eight} - \bar{x}_{Four}}{s_p\sqrt{\frac{1}{n_{Eight}} + \frac{1}{n_{Four}}}} = \frac{15.083 - 10.833}{4.299\sqrt{\frac{1}{12} + \frac{1}{12}}} = 2.422$$

b) $0.010 < p\text{-value} < 0.013$

Step 5 a) $p\text{-value} < 0.013 < \alpha = 0.05$

b) Reject H_o.

∴ There is sufficient evidence at the 5% level of significance to conclude that people who are allowed to sleep for only four hours will score significantly lower on a cognitive skills test than people who are allowed to sleep for eight hours.

Question 5 (7 points)

Ema wants to determine if women like sushi more than men. In a random sample of 320 women, 200 said they liked sushi, and in a random sample of 392 men, 210 said they liked sushi. Construct a 98% confidence interval for the difference in the proportion of men and women who like sushi.

Step 1 Assumptions: $n_W = 320 > 20$ $\quad n_W \hat{p}_W = 200 > 5$ $\quad n_W \hat{q}_W = 120 > 5$

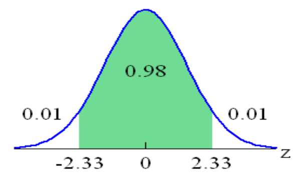
$n_M = 392 > 20$ $\quad n_M \hat{p}_M = 210 > 5$ $\quad n_M \hat{q}_M = 182 > 5$

Samples are independent

Step 2 a) Test statistic: z

b) Level of confidence: $1-\alpha = 0.98$ or $\alpha = 0.02$

Step 3 Point estimate : $\hat{p}_w - \hat{p}_m = \frac{200}{320} - \frac{210}{392} = 0.0893$



Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.01} = 2.33$

b) $E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_w \hat{q}_w}{n_w} + \frac{\hat{p}_m \hat{q}_m}{n_m}} = 2.33 \sqrt{\frac{\frac{200}{320}\frac{120}{320}}{320} + \frac{\frac{210}{392}\frac{182}{392}}{392}} = 0.0861$

c) $(\hat{p}_w - \hat{p}_m) - E < p_w - p_m < (\hat{p}_w - \hat{p}_m) + E$

$0.0893 - 0.0861 < p_w - p_m < 0.0893 + 0.0861$

$0.0031 < p_w - p_m < 0.1754$

Step 5 The 98% confidence interval for the difference in the proportion of men and women who like sushi is 0.3% to 17.5%.

Question 6 (7 points)

Ema wants to determine if women like TV reality shows more than men. In a random sample of 375 women, 195 said they watched a reality show last week, and in a random sample of 421 men, 205 said they watched a reality show last week. Can you conclude, at the 2% level of significance, that women like TV reality shows more than men? Use the classical approach.

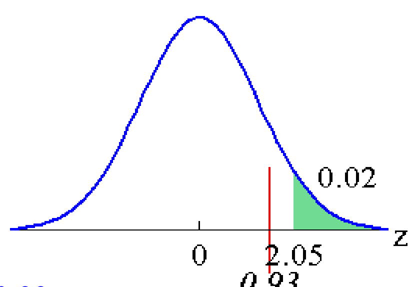
Step 1 Assumptions: $n_W = 375 > 20$ $\quad n_W \hat{p}_W = 195 > 5$ $\quad n_W \hat{q}_W = 180 > 5$

$n_M = 421 > 20$ $\quad n_M \hat{p}_M = 205 > 5$ $\quad n_M \hat{q}_M = 216 > 5$

Samples are independent

Step 2 $H_O: p_W - p_M = 0$

$H_A: p_W - p_M > 0$



Step 3 a) Test statistic: z

b) Right-tailed test $\alpha = 0.02$

c) $z_\alpha = z_{0.02} = 2.05$

Step 4 a) $\hat{p}_p = \frac{195+205}{375+421} = \frac{400}{796} = 0.5025$

b) $z = \frac{\hat{p}_W - \hat{p}_M}{\sqrt{\hat{p}_p \hat{q}_p \left(\frac{1}{n_W} + \frac{1}{n_M}\right)}} = \frac{\frac{195}{375} - \frac{205}{421}}{\sqrt{\frac{400}{796}\frac{396}{796}\left(\frac{1}{375} + \frac{1}{421}\right)}} = 0.93$

Step 5 a) z is not in the critical region

b) Fail to reject H_o.

$\therefore$ There is not sufficient evidence at the 2% level of significance to conclude that women like reality TV shows more then men.

Questions 7 to 9 are to be done using Excel.

For these questions, hand-in a printout of your Excel sheets and copy your Excel work in the Test folder for QM II (W:\Tests\mhuard\QM II\Assignment 3), where your name should be included in the name of the file (for example: Assignment 3 – Your Name). Make sure that your answers are well organized with appropriate labels, and rounded off to an appropriate number of decimal places.

Open the file "Assignment 3 - Data" from my web site, and save it under "Assignment 3 – Your Name". Note that you may have to enable macros to be able to generate the data. If the macros are not enabled (that is, if the data does not appear at the click of the button) then go to - EXCEL OPTIONS – TRUST CENTER – TRUST CENTER SETTINGS – MACRO SETTINGS and choose the ENABLE ALL MACROS option. Note that you may need to close your document and open it again.

Question 7 (6 points)

A psychologist wants to determine if drinking coffee helps with concentration. A random sample of adults was taken where the time to complete a particularly challenging task was measured (in minutes), both before and after drinking a large cup of coffee.

a) Go to the worksheet "Sheet1", rename it appropriately, make the usual heading in cells A1:A4, then click on the "GENERATE DATA" button to get your data.

b) Can we conclude that drinking coffee helps concentration? Use the classical approach along with a 5% level of significance. Assume that the times to complete the task are normally distributed.

8	Time Before (min)	Time After (min)	t-Test: Paired Two Sample for Means			Step 1	Assumptions: Population normally distributed			
9	14.3	9.5				Step 2	H_0:	μ_d =	0	
10	15.9	13.5		Time Before (mi	me After (min)		H_A:	μ_d >	0	
11	14.1	13.9	Mean	15.436	13.9	Step 3	Test statistic: *t* with *df* =			24
12	16.3	13	Variance	6.5424	9.3925		Right-tailed Test		α =	5%
13	12.9	15.3	Observations	25	25		t critical =	1.711		
14	13.2	19.2	Pearson Correlation	-0.2725697		Step 4	t =	1.708		
15	17.4	8	Hypothesized Mean Difference	0		Step 5	*t* is not in the critical region			
16	9.5	19.2	df	24			Fail to reject Ho			
17	21.9	14.6	t Stat	1.7084294			There is insufficient evidence, at the 5% level			
18	14.4	11.9	P(T<=t) one-tail	0.0502302			of significance, to conclude that coffee help			
19	18.9	12.3	t Critical one-tail	1.7108821			concentration.			
20	17.6	9.7	P(T<=t) two-tail	0.1004604						
21	15.6	13.7	t Critical two-tail	2.0638985						
22	15.9	15.9								
23	17.3	12.3								
24	19.3	11.9								

Question 8 (6 points)

A social psychologist was interested in sex differences in the sociability of teenagers. Using the number of good friends as a measure, he compared the sociability of different teenagers.

a) Go to the worksheet “Sheet2”, rename it appropriately, make the usual heading in cells A1:A4, then click on the “GENERATE DATA” button to get your data.

b) Test whether there is any sex difference in the sociability of teenagers. Use the classical approach with a 5% level of significance.

	# friends (Women)	# friends (Men)	t-Test: Two-Sample Assuming Equal Variances						
8						Step 1	Assumptions:	Population normally distributed	
9	6	5						Samples are independent	
10	9	9		# friends (Women)	# friends (Men)			Variances are equal	
11	7	4	Mean	7.8	5.953846	Step 2	H_0:	$\mu_W - \mu_M =$	0
12	9	4	Variance	10.475	9.325962		H_A:	$\mu_W - \mu_M \neq$	0
13	6	11	Observations	65	65	Step 3	Test statistic: t with df =		128
14	10	1	Pooled Variance	9.9004808			Right-tailed test with α =		5%
15	4	10	Hypothesized Mean Difference	0		Step 4	t =	3.345	
16	2	4	df	128			p - value =	0.001080	
17	5	6	t Stat	3.3448868		Step 5	p - value < α		
18	6	4	P(T<=t) one-tail	0.0005402			Reject Ho		
19	2	5	t Critical one-tail	1.6568452			There is sufficient evidence, at the 5% level		
20	0	4	P(T<=t) two-tail	0.0010804			of significance to conclude that there is		
21	5	5	t Critical two-tail	1.9786708			a sex difference in the sociability of teenagers.		
22	12	7							

Question 9 (6 points)

A sociologist is interested in knowing whether men and women watch the same amount of TV. A random sample of men was taken, and another of women, where each person gave the number of hours they watched TV during the previous week. Assume that the standard deviation for females is 4.3 hours and for men 5.2 hours.

a) Go to the worksheet “Sheet3”, rename it appropriately, make the usual heading in cells A1:A4, then click on the “GENERATE DATA” button to get your data.

b) At the 4% level of significance, can you conclude that men and women do not watch the same amount of TV? Use the classical approach.

	Men (hours of TV)	Women (hours of TV)	z-Test: Two Sample for Means							
8						Step 1	Assumptions:	Samples are independent		
9	23	20						n_F =	57	≥ 30
10	14	22		Men (hours of TV)	Women (hours of TV)			n_M =	64	≥ 30
11	16	9	Mean	20.017544	18.203125	Step 2	H_0:	$\mu_M - \mu_W =$	0	
12	23	14	Known Variance	27.04	18.49		H_A:	$\mu_M - \mu_W \neq$	0	
13	20	21	Observations	57	64	Step 3	Test statistic: z			
14	28	23	Hypothesized Mean Difference	0			Two-tailed Test with α =		4%	
15	28	10	z	2.0767879			z critical =	2.054		
16	18	13	P(Z<=z) one-tail	0.0189106		Step 4	z =	2.08		
17	21	20	z Critical one-tail	1.7506861		Step 5	z is in the critical region			
18	22	20	P(Z<=z) two-tail	0.0378211			Reject Ho			
19	17	21	z Critical two-tail	2.0537489			There is insufficient evidence, at the 4% level			
20	17	21					of significance to conclude that men and women			
21	21	15					do not watch the same amount of TV.			
22	23	20								