

MATHEMATICS 360-255-LW

Quantitative Methods II

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Fall 2009

Assignment 2
SOLUTIONS

This assignment is due on **Friday October 23, 2009** at the beginning of the class. Complete solutions are expected.

Part I – To be done by hand

Question 1 (9 points)

To measure anxiety, psychologist will sometimes make use of the Beck Anxiety Inventory test. Assume that the anxiety scores for college or university students are normally distributed with a mean score of 16 and a standard deviation of 6,

- a) What is the probability that a student chosen at random will have a score on the anxiety test higher than 20?

$$\begin{aligned} P(x > 20) &= P(z > 0.67) & z &= \frac{x - \mu}{\sigma} = \frac{20 - 16}{6} = 0.67 \\ &= 1 - 0.7486 \\ &= 0.2514 \end{aligned}$$

- b) What is the probability that a student chosen at random will have a score on the anxiety test between 12 and 15?

$$\begin{aligned} P(12 < x < 15) &= P(-0.67 < z < -0.17) & z_1 &= \frac{x - \mu}{\sigma} = \frac{12 - 16}{6} = -0.67 \\ &= 0.4325 - 0.2514 & z_2 &= \frac{x - \mu}{\sigma} = \frac{15 - 16}{6} = -0.17 \\ &= 0.1811 \end{aligned}$$

- c) What is the probability that in a group of 25 students, the average score on the anxiety test will be between 12 and 15?

$$\begin{aligned} P(12 < \bar{x} < 15) &= P(-3.33 < z < -0.83) & z_1 &= \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{12 - 16}{\frac{6}{\sqrt{25}}} = -3.33 \\ &= 0.2033 - 0.0004 & z_2 &= \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{15 - 16}{\frac{6}{\sqrt{25}}} = -0.83 \\ &= 0.2029 \end{aligned}$$

Question 2 (3 points)

The life of an Ipod is normally distributed with a mean of 6 years and a standard deviation of 1.5 years. The manufacturer will replace any defective Ipod's free of charge while it is under guarantee. For how many years should an Ipod be guaranteed if the manufacturer does not want to replace more than 5% of them?

$$\text{Area to the left} = 0.05$$

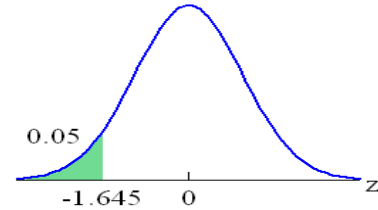
$$z = -1.645$$

$$x = \mu + z\sigma$$

$$= 6 - 1.645 \cdot 1.5$$

$$= 3.53$$

Thus the guarantee should be for 3 ½ years.

**Question 3** (3 points)

Sarah discovers that the amount of time it takes her to drive to work is normally distributed with a mean of 35 minutes and a standard deviation of 5 minutes. At what time should Sarah leave her home so that she has a 99% chance of arriving at work before 9 A.M.?

$$\text{Area to the left} = 0.99$$

$$z = 2.33$$

$$x = \mu + z\sigma$$

$$= 35 + 2.33 \cdot 5$$

$$= 46.65$$

Thus she should leave 47 minutes before 9 A.M., hence she should leave at 8:13 A.M.

Question 4 (8 points)

It is a well known fact that eighty percent of students at St. Lawrence think QM II is the most interesting course. If 120 students are chosen at random, find the probability that

- a) Between 90 and 105 think QM II is the most interesting course.

$$\text{Assumptions: } np = 120 \cdot 0.80 = 96 > 5$$

$$\mu = np = 96$$

$$nq = 120 \cdot 0.20 = 24 > 5$$

$$\sigma = \sqrt{npq} = \sqrt{19.2} = 4.38$$

$$\text{Thus } B(120, 0.80) \sim N(96, 19.2)$$

$$P(90 < r < 105) = P(90.5 < x < 104.5)$$

$$= P(-1.26 < z < 1.94)$$

$$= 0.9738 - 0.1038$$

$$= 0.8700$$

$$z_1 = \frac{x - \mu}{\sigma} = \frac{104.5 - 96}{4.38} = 1.94$$

$$z_2 = \frac{90.5 - 96}{4.38} = -1.26$$

- b) Over 100 students think QM II is the most interesting course.

$$P(r > 100) = P(x > 100.5)$$

$$= P(z > 1.03)$$

$$= 1 - 0.8485$$

$$= 0.1515$$

$$z = \frac{x - \mu}{\sigma} = \frac{100.5 - 96}{4.38} = 1.03$$

Question 5 (8 points)

In a test on the perception of time, a psychologist asked 40 adults chosen at random to estimate a fixed time interval of 20 seconds. An average of 21.45 seconds was obtained. Previous research has shown that $\sigma = 4.057$ seconds. Construct a 97% interval for the mean estimate of time.

- Step 1 Assumptions: $n = 40 \geq 30$
- Step 2 a) Test statistic: z
b) Level of confidence: $1 - \alpha = 0.97$ or $\alpha = 0.03$
- Step 3 Point estimate: $\bar{x} = 21.45$ seconds
- Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.015} = 2.17$
b) $E = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 2.17 \frac{4.057}{\sqrt{40}} = 1.39$
c) $\bar{x} - E < \mu < \bar{x} + E$
 $21.45 - 1.39 < \mu < 21.45 + 1.39$
 $20.06 < \mu < 22.84$
- Step 5 The 97% confidence interval for the mean estimate of time is 20.06 to 22.84 seconds.

Question 6 (8 points)

A college psychologist wants to determine the number of hours of sleep CEGEP students get each night. She takes a random sample of students, and asks each of them for the number of hours they slept last night. Here are the results.

8.1	9.6	6.9	7.5	7.2	7.9	7.7	7.4	7.5	7.5	7.6
8.1	8.2	7.5	7.6	6.6	6.4	8.7	6.5	8.8	8.2	8.5

Assume that the number of hours of sleep students get is normally distributed. Construct a 99% interval for the mean number of hours of sleep CEGEP students get each night.

- Step 1 Assumptions: The sampled population is normally distributed
- Step 2 a) Test statistic: t with $df = n - 1 = 21$
b) Level of confidence: $1 - \alpha = 0.99$ or $\alpha = 0.01$
- Step 3 Point estimate: $\bar{x} = 7.727$ hours
- Step 4 a) $t_{(df, \frac{\alpha}{2})} = t_{(21, 0.005)} = 2.831$
b) $E = t_{(df, \frac{\alpha}{2})} \frac{s}{\sqrt{n}} = 2.831 \frac{0.781}{\sqrt{22}} = 0.471$
a) $\bar{x} - E < \mu < \bar{x} + E$
 $7.727 - 0.471 < \mu < 7.727 + 0.471$
 $7.26 < \mu < 8.20$
- Step 5 The 99% confidence interval for the mean is number of hours of sleep CEGEP students get each night is 7.26 hours to 8.20 hours

Question 7 (8 points)

A random sample of Canadians was taken, where each was asked if he/she would ever consult a psychologist. Here are the results.

Yes	No	Yes	Yes	Yes	No	No	Yes	Yes
No	Yes	No	Yes	No	Yes	Yes	No	No
Yes	Yes	No	No	Yes	Yes	Yes	No	No
No	No	Yes	No	Yes	Yes	No	Yes	No
Yes	No	No	No	No	Yes	Yes	Yes	No
Yes	Yes	No	Yes	No	No	Yes	No	Yes
Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Yes	No	No	Yes	Yes	No	Yes	No	Yes
Yes	No	Yes	No	No	No	Yes	Yes	No

Construct a 92% confidence interval for the proportion of Canadians who would ever consult a psychologist.

Step 1 Assumptions: $n = 81 > 20$ $n\hat{p} = 46 > 5$ and $n\hat{q} = 35 > 5$

Step 2 a) Test statistic: z

b) Level of confidence: $1 - \alpha = 0.92$ or $\alpha = 0.08$

Step 3 Point estimate: $\hat{p} = \frac{x}{n} = \frac{46}{81} = 0.5679$

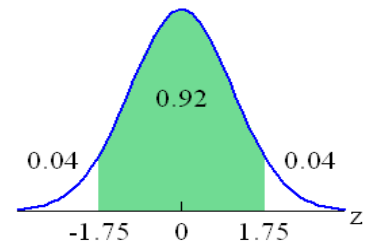
Step 4 a) $z_{\frac{\alpha}{2}} = z_{0.04} = 1.75$

b) $E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.75 \sqrt{\frac{46 \cdot 35}{81 \cdot 81}} = 0.0963$

c) $\bar{x} - E < p < \bar{x} + E$

$$0.5679 - 0.0963 < p < 0.5679 + 0.0963$$


$$0.4716 < p < 0.6642$$



Step 5 The 92% confidence interval for the proportion of Canadians who would ever consult a psychologist is 47.2% to 66.4%

Part II – To be done with Excel

For these questions, hand-in the printouts of your Excel sheets and copy your Excel work in the Test folder for Business Statistics (W:\Tests\mhuard\QM II\Assignment 2), where your name should be included in the name of the file (for example: Assignment 2 – Your Name). Make sure that your answers are well organized with appropriate labels, and rounded off to an appropriate number of decimal places.

Open the file “Data – Assignment 2” from my web site, and save it under “Assignment 2 – Your Name”. Note that you may have to enable macros to be able to generate the data (for questions 9). If the macros are not enabled (that is, if the data does not appear at the click of the button) then go to  - EXCEL OPTIONS – TRUST CENTER – TRUST CENTER SETTINGS – MACRO SETTINGS and choose the ENABLE ALL MACROS option. Note that you may need to close your document and open it again.

Question 8 (5 points)

Go to the worksheet “Question 8” and make the usual heading in cells A1:A4. Verify the answers to questions 1 to 3 with Excel. That is, get Excel to do all your calculations using the functions NORMDIST and NORMINV.

7	Question 1								
8	a)	$P(x > 20) =$	0.2525						
9	b)	$P(12 < x < 15) =$	0.1813						
10	c)	$P(12 < \bar{x} < 15) =$	0.2019						
11									
12	Question 2								
13		x =	3.53 years						
14									
15									
16	Question 3								
17		x =	46.63 minutes						
18		Time:	8:13 A.M.						
19									

Question 9 (8 points)

An academic counselor decided to investigate the length of time students take to get to school in the morning. For this, she took a random sample of 44 students, and noted for each the time (in minutes) it took them to get to school.

- a) Go to the worksheet “Question 9”, make the usual heading in cells A1:A4, then click on the “GENERATE DATA” button to get your data.
- b) Construct a 98% confidence interval for the mean length of time students take to get to school.

5									
6		Generate Data							
7									
8	Time (min)		<i>Time (min)</i>		Step 1	Assumptions: n =	60	≥ 30	
9	44				Step 2	Test statistic: t with df =	59		
10	52	Mean	45.4833333			1 - α =	98%		
11	20	Standard Error	2.12590847		Step 3	Mean	45.48		
12	47	Median	44		Step 4	E =	5.08		
13	22	Mode	48		Step 5	The 98% confidence interval for the mean			
14	48	Standard Deviation	16.4672162			time students take to get to school is			
15	26	Sample Variance	271.169209			40.40 min to	50.57 min		
16	60	Kurtosis	-0.3629876						
17	28	Skewness	0.39914792						
18	34	Range	69						
19	71	Minimum	13						
20	27	Maximum	82						
21	73	Sum	2729						
22	65	Count	60						
23	40	Confidence Level(98.0%)	5.0835336						
24	19								