

MATHEMATICS 360-255-LW

Quantitative Methods II

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Fall 2009

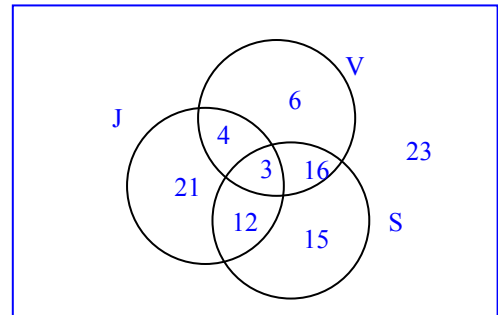
Assignment 1 SOLUTIONS

This assignment is due on **Friday September 11, 2009** at the beginning of the class. Complete solutions are expected.

Question 1 (4 points)

The dean of a college wants to look at student involvement in extracurricular activities. A sample of 100 students was taken, where each was asked whether they have a part-time job, do volunteer work or play a sport. The results were as follows:

- 40 have a part-time job
- 29 do volunteer work
- 46 play a sport
- 7 have a part-time job and do volunteer work
- 19 do volunteer work and play a sport
- 15 have a part-time job and play a sport
- 3 do all three



What is the probability a student chosen at random from the survey

- a) only has a part-time job; $\frac{21}{100}$
- b) is involved in at least one of the three extracurricular activities. $\frac{77}{100}$

Question 2 (4 points)

Suppose the probability that it will rain on any given day is 40%.

- a) What is the probability that it will rain for three consecutive days?

$$\begin{aligned} P(S_1 S_2 S_3) &= P(S_1) P(S_2) P(S_3) \\ &= 0.4 \cdot 0.4 \cdot 0.4 \\ &= 0.064 \end{aligned}$$

- b) What is the probability that it will rain only once in the next three days?

$$\begin{aligned} P(S_1 \bar{S}_2 \bar{S}_3 \text{ or } \bar{S}_1 S_2 \bar{S}_3 \text{ or } \bar{S}_1 \bar{S}_2 S_3) &= P(S_1) P(\bar{S}_2) P(\bar{S}_3) + P(\bar{S}_1) P(S_2) P(\bar{S}_3) + P(\bar{S}_1) P(\bar{S}_2) P(S_3) \\ &= 0.4 \cdot 0.6 \cdot 0.6 + 0.6 \cdot 0.4 \cdot 0.6 + 0.6 \cdot 0.6 \cdot 0.4 \\ &= 0.432 \end{aligned}$$

Question 3 (2 points)

A psychologist only has time to see six patients during the day (appointments being at 9, 10, 11 in the morning and 2, 3 and 4 in the afternoon). How many different schedules are there if the six patients are chosen from a list of 15 patients?

$$P_6^{15} = 3\,603\,600$$

Question 4 (8 points)

A poker hand is a subset of five cards drawn from a deck of 52 cards. Find the probability that the poker hand has

a) Only diamonds.

$$\frac{C_5^{13}}{C_5^{52}} = \frac{1287}{2598960} = \frac{33}{66640} \approx 0.0004952$$

b) Two queens and three aces.

$$\frac{C_2^4 C_3^4}{C_5^{52}} = \frac{24}{2\,598\,960} = \frac{1}{108\,290} \approx 0.000009234$$

c) Exactly two diamonds, two clubs and a heart.

$$\frac{C_2^{13} C_2^{13} C_1^{13}}{C_5^{52}} = \frac{78 \cdot 78 \cdot 13}{2\,598\,960} = \frac{79092}{2\,598\,960} = \frac{507}{16660} \approx 0.03043$$

d) Exactly four diamonds.

$$\frac{C_4^{13} C_1^{39}}{C_5^{52}} = \frac{715 \cdot 39}{2\,598\,960} = \frac{143}{13328} \approx 0.01073$$

Question 5 (9 points)

A survey questioned residents of Quebec City on whether or not they went to see a movie at the theaters this summer. The following results were obtained.

	Went to see a movie	Did not go see a movie	
Under 25	20	30	50
Between 25 and 45	45	40	85
Over 45	35	80	115
	100	150	250

Suppose a person is selected at random from the survey. Let us use the following notation for events: W = went to see a movie, U = under 25, B = between 25 and 45, O = over 45.

a) Find $P(W)$ and $P(W|B)$.

$$P(W) = \frac{100}{250} = \frac{2}{5} \qquad P(W|B) = \frac{45}{85} = \frac{9}{17}$$

b) Are the events W and B independent? Explain.

$$\text{No since } P(W) = \frac{100}{250} = \frac{2}{5} \neq P(W|B) = \frac{45}{85} = \frac{9}{17}$$

c) Are the events W and U independent? Explain.

$$\text{Yes since } P(W) = \frac{100}{250} = \frac{2}{5} = P(W|U) = \frac{20}{50} = \frac{2}{5}$$

d) Find $P(W \text{ and } B)$. $P(W \text{ and } B) = \frac{45}{250} = \frac{9}{50}$

e) Are the events W and B mutually exclusive? Explain.

$$\text{No since } P(W \text{ and } B) = 0.18 \neq 0$$

f) Find $P(W \text{ or } B)$.

$$P(W \text{ or } B) = P(W) + P(B) - P(W \text{ and } B) = \frac{100}{250} + \frac{85}{250} - \frac{45}{250} = \frac{14}{25}$$

Question 6 (8 points)

A psychotherapy clinic offers two seminars to help with depression, one using an Existential-Humanistic based approach (EH), and the other a Cognitive-Behavioral approach (CB). It has been observed that 30% of people who participated in the seminar with the EH approach found the seminar helped, while 50% of those with the CB approach found the seminar helpful. Last month, 180 people participated in the seminar with the EH approach, while 120 participated with the CB approach.

- a) What is the probability a person who took the seminar had the CB approach?

$$P(CB) = \frac{120}{120+180} = \frac{120}{300} = \frac{2}{5} = 0.4$$

- b) What is the probability a person found the seminar helpful?

$$\begin{aligned} P(H) &= P(CBH \text{ or } EHH) = P(CB)P(H|CB) + P(EH)P(H|EH) \\ &= \frac{120}{300} \cdot 0.5 + \frac{180}{300} \cdot 0.3 \\ &= \frac{114}{300} \\ &= \frac{19}{50} \end{aligned}$$

- c) What is the probability a person found the seminar helpful or had the CB approach?

$$\begin{aligned} P(H \text{ or } CB) &= P(H) + P(CB) - P(CBH) \\ &= \frac{19}{50} + \frac{2}{5} - \frac{120}{300} \cdot 0.5 \\ &= \frac{174}{300} \\ &= \frac{29}{50} \end{aligned}$$

- d) What is the probability a person had the EH approach if he found the seminar helpful?

$$\begin{aligned} P(EH|H) &= \frac{P(EHH)}{P(H)} \\ &= \frac{\frac{180}{300} \cdot 0.3}{\frac{174}{300}} \\ &= \frac{54}{174} \\ &= \frac{6}{29} \end{aligned}$$

Question 7 (5 points)

In Quebec, 80% of the population has French as a mother tongue, 8% has English and 12% has another language. Also, 25% of Quebecers with French as a mother tongue speak another language, 95% of Quebecers with English as a mother tongue speak another language and 85% of Quebecers who have neither French nor English as a mother tongue speak another language. A Quebecer is selected at random.

- a) What is the probability that he speaks a second language?

$$\begin{aligned} P(S) &= P(F)P(S|F) + P(E)P(S|E) + P(O)P(S|O) \\ &= (0.8)(0.25) + (0.08)(0.95) + (0.12)(0.85) \\ &= 0.378 \end{aligned}$$

- b) What is the probability that his mother tongue is French given that he speaks a second language?

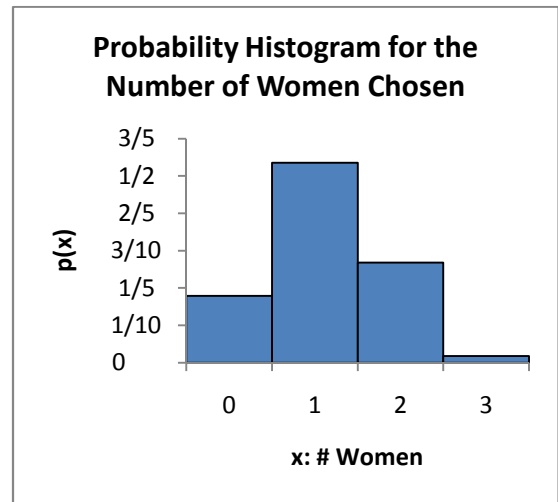
$$\begin{aligned} P(F|S) &= \frac{P(F \text{ and } S)}{P(S)} = \frac{P(F)P(S|F)}{P(S)} \\ &= \frac{0.8 \cdot 0.25}{0.378} = 0.529 \end{aligned}$$

Question 8 (11 points)

A psychologist has eight patients who suffer from schizophrenia, five men and three women. A group of three is to be selected at random to take part in a new therapy. Let x be the number of women chosen in the group.

- a) Find the probability distribution for x .

Probability Distribution for the Number of Women Chosen	
x : # Women	$p(x)$
0	$\frac{C_3^5}{C_3^8} = \frac{5}{28}$
1	$\frac{C_1^3 C_2^5}{C_3^8} = \frac{15}{28}$
2	$\frac{C_2^3 C_1^5}{C_3^8} = \frac{15}{56}$
3	$\frac{C_3^3}{C_3^8} = \frac{1}{56}$



- b) Draw a histogram for the probability distribution.
 c) Find the mean of this probability distribution.

$$\mu = \sum xp(x) = 0 \cdot \frac{5}{28} + 1 \cdot \frac{15}{28} + 2 \cdot \frac{15}{56} + 3 \cdot \frac{1}{56} = \frac{9}{8} \text{ women}$$

- d) Find the standard deviation for this probability distribution.

$$\begin{aligned} \sigma^2 &= \sum x^2 p(x) - \mu^2 = \left(0^2 \cdot \frac{5}{28} + 1^2 \cdot \frac{15}{28} + 2^2 \cdot \frac{15}{56} + 3^2 \cdot \frac{1}{56}\right) - \left(\frac{9}{8}\right)^2 = \frac{99}{56} - \frac{81}{64} = \frac{225}{448} \text{ women}^2 \\ \sigma &= \sqrt{\sigma^2} = \sqrt{\frac{1}{2}} \approx 0.7087 \text{ women} \end{aligned}$$

Question 9 (9 points)

An insurance company has found that the probability that a life insurance applicant will qualify at a special rates is 0.2. Last week, there were 8 applicants for life insurance.

- a) What is the probability that exactly three of these applicants will qualify at a special rate?

$$B(8, 0.20)$$

$$P(3) = C_3^8 (0.20)^3 (0.80)^5 = 0.1468$$

- b) What is the probability that at least one of the applicants will qualify at a special rate?

$$P(r \geq 1) = 1 - P(0)$$

$$= 1 - C_0^8 (0.80)^0 (0.20)^8$$

$$= 0.8322$$

- c) How many of the applicants do you expect to qualify at a special rate?

$$\mu = np = 0.2 \cdot 8 = 1.6 \text{ applicants}$$

- d) Using Excel, construct the probability distribution, along with the probability histogram, for the given distribution. Make sure that your answers are well organized with appropriate labels, and rounded off (usually to four significant figures). Also, your name should appear at the top of the Spreadsheet. Hand-in the printout of your Excel sheet and copy your Excel work in the Test folder for QMII (W:\Tests\mhuard\QM II\Assignment 1)).

