

QUANTITATIVE METHODS

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Review Problems

1. Sarah wants to determine the mean number of hours per week a student at St. Lawrence studies. To accomplish this, she passed a questionnaire to 10 students chosen at random from their registration number. The results were:

4 12 7 10 15 5 2 22 15 18

Find the following:

- | | |
|-----------------------------|----------------------------------|
| a) the mean | b) the median |
| c) the mode | d) the range |
| e) the sample variance | f) the standard deviation |
| g) Q_1 | h) Q_3 |
| i) Coefficient of Variation | j) Draw a box and whisker graph. |
2. Here is a frequency distribution showing the ages of 121 randomly selected people who have a bachelor's degree or higher.

Ages	Frequency
18-24	8
25-34	32
35-44	35
45-54	23
55-64	11
65-84	9
85-104	3

Use this frequency distribution to estimate

- the mean
- the variance
- the standard deviation

3. The following data on the stress scores before a QM test and the QM test scores for seven students are:

Stress Score	6.5	4.0	2.5	7.2	8.1	3.4	5.5
Test score	81	96	93	68	63	84	71

- Find the slope of the least-squares line.
 - Find the intercept of the least-squares line.
 - Find the equation of the least squares line.
 - Find the coefficient of correlation r .
 - Find the coefficient of determination.
 - If a student obtained a stress score of 7.0, what is the expected test score?
 - Draw a scatter diagram.
4. A box contains 3 marbles, a red, a blue and a green marble. If two marbles are picked at random, what is the probability that
- one is red and the other is green?
 - both are red?
5. Two dice are rolled, where one is black and the other is white. Find the following probabilities.
- $P(\text{white die is an odd number})$
 - $P(\text{sum is } 6)$
 - $P(\text{both dice show odd numbers})$
 - $P(\text{number on black die is larger than number on white die})$
6. Two thousand randomly selected adults were asked if they think they are financially better off than their parents. The following table gives the two-way classification of the responses based on the education levels of the persons included in the survey and whether they are financially better off, the same, or worse off than their parents.

	Education Level		
	High school or less	CEGEP	More than CEGEP
Better off	140	450	420
Same	60	250	110
Worse off	200	300	70

Suppose one adult is selected at random from these 2000 adults. Find the following probabilities

- $P(\text{better off})$
- $P(\text{better off and CEGEP})$
- $P(\text{better off or CEGEP})$
- $P(\text{better off given CEGEP})$
- Are the events better off and CEGEP independent?
- Are the events better off and CEGEP mutually exclusive?

7. A single card is drawn from a deck of 52 cards. What is the probability that the card is
- a king of diamonds?
 - a face card?
 - not an ace?
 - a king or a diamond?
 - a red face card?
8. A consume agency surveyed all 2500 families living in a small town to collect data on the number of television sets owned by them. The following table lists the frequency distribution of the data collected by this agency.

Number of TV sets owned	0	1	2	3	4
Number of families	120	970	730	410	270

- Construct a probability distribution table for the number of television sets owned by these families.
 - Draw a histogram.
 - Find $P(x > 2)$
 - Find $P(x \leq 1)$
 - Find $P(1 \leq x \leq 2)$
 - How many TV sets do you expect a family chosen at random to have?
 - Find the standard deviation of this distribution.
9. Suppose that the time taken to run a road race is normally distributed with a mean of 195 minutes and a standard deviation of 21 minutes. If a runner is selected at random, what is the probability that this runner will complete the this road race
- in less than 150 minutes
 - in 205 to 245 minutes
10. Express Courier Service has found that the delivery times for packages are normally distributed with mean 14 hours and standard deviation 2 hours.
- For a package selected at random, what is the probability that it will be delivered in 18 hours or less?
 - For a package selected at random, what is the probability that it will be delivered in more than 15 hours?
 - For a package selected at random, what is the probability that it will be delivered in between 10 and 20 hours?
 - What should the guaranteed delivery time on all packages be in order to be 95% sure that a given package will be delivered within this time?
11. Quick Start Company makes 12-volt car batteries. After many years of product testing, the company knows that the average life of a Quick Start battery is normally distributed with a mean of 45 months and a standard deviation of 8 months.
- If Quick Start guarantees a full refund on any battery that fails within the 36-month period after purchase, what percentage of its batteries will the company expect to replace?
 - If Quick Start does not want to replace more than 10% of its batteries under the full-refund guarantee policy, for how long should the company guarantee the batteries (to the nearest month)?

12. The ages of all university students follow a distribution with a mean of 23 years and a standard deviation of 4 years. Find the probability that the mean age for a random sample of 36 students would be
- between 22 and 24 years
 - more than 22 years
13. A new muscle relaxant is available. Researchers at the firm developing the relaxant have done studies indicating that the time lapse between administration of the drug and beginning effects of the drug are normally distributed with mean 38 minutes and standard deviation 5 minutes.
- The drug is administered to one patient selected at random. What is the probability that the time it takes to go into effect is 35 minutes or less?
 - The drug is administered to a random sample of 10 patients. What is the probability that the average time before it is effective for all 10 patients is 35 minutes or less?
14. Thirty randomly selected college students were asked how many cavities they had. These students had a mean of 3.2 cavities with a standard deviation of 1.65 cavities. Construct a 99% confidence interval for the mean number of cavities for all college students.
15. The administration at a college wishes to answer the question “How far (one way) does the average community student commute to college each day?” A random sample of 100 commuting students was identified, and the one-way distance each commuted was obtained. The resulting sample mean distance was 18.5 kilometers with a standard deviation of 9.1 kilometers. Make a 97% confidence interval for the mean one-way commuting distance of the college students.
16. A high-tech company wants to estimate the mean number of years of college education its employees have completed. A good estimate of the standard deviation for the mean number of years of college is 1.2. How large a sample needs to be taken to estimate μ to within 0.5 of a year with 99% confidence?
17. A department store manager wants to estimate at a 90% confidence level the mean amount spent by all customers at this store. From an earlier study, the manager knows that the standard deviation of amounts spent by customers at this store is \$27. What sample size should he choose so that the estimate is within 3\$ of the population mean?
18. To determine the views of students at SLC on whether an extremist hate group should be given a permit to demonstrate, a seven-point attitude scale (1 = strongly opposed through 7 = strongly favor) was administered to a random sample of 15 students. This survey yielded a sample mean of 2.1 and a standard deviation of 1.5. Assuming that the scores are approximately normally distributed, construct a 95% confidence interval for the mean population score.

19. Suppose a researcher wanted to examine the extent of cooperation in kindergarten children. To do so, she unobtrusively observes a group of children at play for 30 minutes and notes the number of cooperative acts engaged in by each child. Here are the number of cooperative acts exhibited by each child:

1 5 2 3 4 1 2 2 4 3

Construct a 90% confidence interval for the mean number of cooperative acts exhibited by children, assuming that the number of cooperative acts exhibited by children is normally distributed.

20. A random sample of 250 registered voters revealed that 63 of them feel that education is the most important issue when deciding on a candidate. Construct a 90% confidence interval for the proportion of all registered voters who feel that education is the most important issue when deciding on a candidate.
21. A mail-order company promises its customers that the products ordered will be mailed within 72 hours after an order is placed. The quality control department at the company checks from time to time to see if this promise is fulfilled. Recently the quality department took a sample of 50 orders and found that 42 of them were mailed within 72 hours of the placement of the orders. Construct a 98% confidence interval for the percentage of all orders that are mailed within 72 hours of their placement.
22. A consumer agency wants to estimate the proportion of all drivers who wear seat belts while driving. Assume that a preliminary study has shown that 76% of drivers wear seat belts while driving. How large should the sample size be so that the 99% confidence interval for the population proportion has a maximum error of 3%?
23. A researcher wants to determine what proportion of all high school students have Internet access at home. He has no idea what the sample proportion will be. How large a sample is required to be 95% sure that the sample proportion is off by no more than 5%?
24. A student wants to determine what percentage of college students smoke. How large a sample should she take to be 90% confident that her sample proportion is off by no more than 4.5%?
25. In order to investigate whether conformity changes from one generation to the next, a test that measures an individual's nonconformity rating was designed by a sociologist. The larger the score on the test, the more nonconforming the individual. In 1970, the average score was 153. To see if a change occurred, the test was given to 45 randomly selected individuals in 2000 and the mean score was found to be 161 with a standard deviation of 17. Test at the 3% level of significance whether nonconformity has increased between 1970 and 2000. Try with both approaches, the classical and the p -value.

26. A sample of 225 parents were asked how much time they spent per week on school work or school-related activities. This sample produced a mean of 5.6 hours per week, with a standard deviation of 4.4 hours. At the 1% level of significance, test the claim that the mean number of hours spent by parents on school work or school-related activities is 5 hours per week. Try with both approaches, the classical and the p -value.
27. A telephone company claims that the mean duration of all long-distance phone calls made by its residential customers is 10 minutes. A random of 100 long-distance calls made by its residential customers taken from the records of this company showed that the mean duration of calls for this sample is 9.0 minutes with a standard deviation of 5.2 minutes. Test, at the 2% level of significance whether the mean duration of all long-distance calls is less than 10 minutes. Try with both approaches, the classical and the p -value.
28. At the 1% level of significance, test the claim that the hours worked by college students is greater than 15 hours per week. A random sample of 25 students produced a sample mean of 20.83 hours per week with a standard deviation of 14.20 hours per week. Assume that the number of hours worked by college students is normally distributed.
29. How many pair of shoes do female college students own? A random sample of 15 female college students produced a sample mean of 8.7 pairs of shoes, with a standard deviation of 0.85 pairs. Use these data to test the claim that the mean number of pairs of shoes owned by female college students is less than 10 at the 5% level of significance. Assume that the number of pair of shoes owned by female students is normally distributed.
30. A past study claims that adult Canadians spend an average of 18 hours a week on leisure activities. A researcher wanted to test this claim. She took a random sample of 10 adults and asked them about the time they spend per week on leisure activities. A mean of 16.3 hours per week with a standard deviation of 3.8 hours per week was obtained. Is this sufficient evidence, at the 10% level of significance, to conclude that the claim is no longer valid? Assume that the time spent on leisure activities by all adults is normally distributed.
31. A mail-order company claims that 60% of all orders are mailed within 48 hours. From time to time the quality control department at the company checks to see if this promise is fulfilled. Recently, the quality control department at this company took a sample of 400 orders and found that 212 of them were mailed within 48 hours of the placement of the orders. Testing at the 3% significance level, can you conclude that less then 60% of all orders are mailed within 48 hours? Try with both approaches, the classical and the p -value.
32. Thirty-five percent of the physicians in Canada were women in 1995. A recent sample of 300 Canadian physicians found that 147 of them were women. Using a 2% significance level, can you conclude that the current percentage of physicians in Canada is higher than it was in 1995? Try with both approaches, the classical and the p -value.

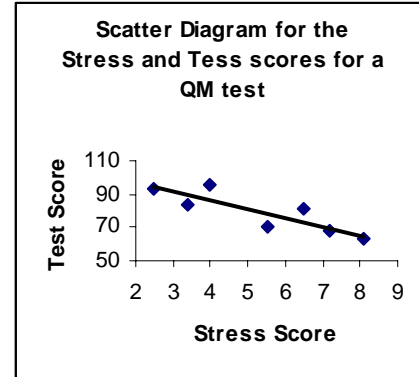
33. In 1995, 32% of Canadian households owned a personal computer. In a recent sample of 850 Canadian households, 305 own personal computers. Test at the 2% level of significance whether the current percentage of all Canadian households who own personal computers is different from 32%.
34. The manager of an assembly process wants to determine whether the number of defective articles manufactured depends on the day of the week the articles are produced. She collected the following information.

	Monday	Tuesdays	Wednesday	Thursday	Friday
Nondefective	85	90	95	95	90
Defective	15	10	5	5	10

Is there sufficient evidence to reject the hypothesis that the number of defective articles is independent of the day of the week on which they are produced? Use a 5% level of significance.

ANSWERS

1. a) 11 hour b) 11 hours c) 15 hours d) 20 hours e) 42.89 hours²
 f) 6.54 hours g) 5 hours h) 15 hours i) 59.5%
2. a) 43.3 years b) 258.1 years² c) 16.1 years
3. a) -5.207
 b) 107.1
 c) $y = -5.207x + 107.1$
 d) -0.861
 e) 74.1%
 f) 70.7

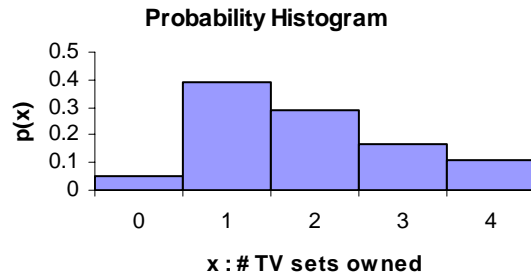


4. a) $\frac{1}{3}$ b) 0
5. a) $\frac{1}{2}$ b) $\frac{5}{36}$ c) $\frac{1}{4}$ d) $\frac{5}{12}$
6. a) $\frac{101}{200}$ b) $\frac{9}{40}$ c) $\frac{39}{50}$ d) $\frac{9}{20}$
 e) No since $P(\text{better off}) = \frac{101}{200} \neq P(\text{better off given CEGEP}) = \frac{9}{20}$
 f) No since $P(\text{better off and CEGEP}) = \frac{9}{40} \neq 0$
7. a) $\frac{1}{52}$ b) $\frac{4}{13}$ c) $\frac{3}{13}$ d) $\frac{3}{26}$ e) $\frac{12}{13}$

8. c) 0.272
 d) 0.436
 e) 0.680
 f) 1.896
 g) 1.08

Probability Distribution

x	$p(x)$
0	0.048
1	0.388
2	0.292
3	0.164
4	0.108



9. a) 0.0162 b) 0.3069
10. a) 0.9772 b) 0.3085 c) 0.9759 d) 17.3 hours
11. a) 13% b) 35 months
12. a) 0.8664 b) 0.9332
13. a) 0.2743 b) 0.0287
14. The 99% confidence interval for the mean number of cavities for all college students is 2.42 to 3.98 cavities
15. The 97% confidence interval for the mean one-way commuting distance of college students is 16.53 to 20.47 kilometers.
16. 39 employees
17. 220 customers
18. The 95% confidence interval for the mean population score is 1.27 to 2.93.
19. The 90% confidence interval for the mean number of cooperative acts exhibited by children is 1.92 to 3.48.

20. The 90% confidence interval for the proportion of all registered voters who feel that education is the most important issue when deciding on a candidate is 20.68% to 28.72%.
21. The 98% confidence interval for the percentage of all orders that are mailed within 72 hours of their placement is 71.9% to 96.1%.
22. At least 1350 drivers
23. At least 385 high school students
24. At least 335 college students
25. $H_0 : \mu = 153$ $z_c = 1.88$ $p\text{-value} = 0.0009$ Reject H_0
 $H_A : \mu > 153$ $z = 3.16$
26. $H_0 : \mu = 5$ hours $z_c = \pm 2.58$ $p\text{-value} = 0.0404$ Fail to reject H_0
 $H_A : \mu \neq 5$ hours $z = 2.05$
27. $H_0 : \mu = 10$ minutes $z_c = -2.05$ $p\text{-value} = 0.0274$ Fail to reject H_0
 $H_A : \mu < 10$ minutes $z = -1.92$
28. $H_0 : \mu = 15$ hours $t_c = 2.492$ Fail to reject H_0
 $H_A : \mu > 15$ hours $t = 2.05$
29. $H_0 : \mu = 10$ shoes $t_c = -1.71$ Reject H_0
 $H_A : \mu < 10$ shoes $t = -5.92$
30. $H_0 : \mu = 18$ hours per week $t_c = \pm 1.833$ Fail to reject H_0
 $H_A : \mu \neq 18$ hours per week $t = -1.41$
31. $H_0 : p = 0.6$ $z_c = -1.88$ $p\text{-value} = 0.0021$ Reject H_0
 $H_A : p < 0.6$ $z = -2.86$
32. $H_0 : p = 0.35$ $z_c = 2.05$ $p\text{-value} = 0.000$ Reject H_0
 $H_A : p > 0.35$ $z = 5.08$
33. $H_0 : p = 0.32$ $z_c = \pm 2.33$ $p\text{-value} = 0.0150$ Reject H_0
 $H_A : p \neq 0.32$ $z = 2.43$
34. H_0 : The number of defective articles is independent of the day of the week on which they are produced gender.
 H_A : It is dependent.
 $\chi^2 = 8.548$ critical value: $\chi_c^2 = 9.49$ Fail to reject H_0