

QUANTITATIVE METHODS

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Assignment 14

SOLUTIONS

This assignment is due **Wednesday March 23, 2005**. You must send your Excel work for questions 5 and 6 by *e-mail*.

Question 1 (6 points)

Assuming that x has a normal distribution with a mean of 7.1 and a standard deviation of 3.2, find the following probabilities.

a) $P(2 \leq x \leq 9)$

$$\begin{aligned} P(2 \leq x \leq 9) &= P(-1.59 \leq z \leq 0.59) \\ &= 0.7224 - 0.0559 \\ &= 0.6665 \end{aligned}$$

$$\begin{aligned} z &= \frac{x - m}{s} = \frac{2 - 7.1}{3.2} = -1.59 \\ z &= \frac{x - m}{s} = \frac{9 - 7.1}{3.2} = 0.59 \end{aligned}$$

b) $P(x \geq 10)$

$$\begin{aligned} P(x \geq 10) &= P(z \geq 0.91) \\ &= 1 - 0.8186 \\ &= 0.1814 \end{aligned}$$

$$z = \frac{x - m}{s} = \frac{10 - 7.1}{3.2} = 0.91$$

c) $P(x < 4)$

$$\begin{aligned} P(x < 4) &= P(z < -0.97) \\ &= 0.1660 \end{aligned}$$

$$z = \frac{x - m}{s} = \frac{4 - 7.1}{3.2} = -0.97$$

Question 2 (4 points)

Assuming that x has a normal distribution with a mean of 42.6 and a standard deviation of 6.5,

- a) Find the x value such that 15% of the area below the curve lies to the right of x .

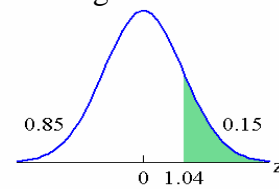
Area to the left of z is 0.85

$$z = 1.04$$

$$x = m + zS$$

$$= 42.6 + 1.04 \cdot 6.5$$

$$= 49.36$$



- b) Find the x values such that 92% of the area below the curve lies between x_1 and x_2 .

Area to the left of z_1 is 0.04

$$z_1 = -1.75$$

$$x_1 = m + z_1S$$

$$= 42.6 - 1.75 \cdot 6.5$$

$$= 31.23$$

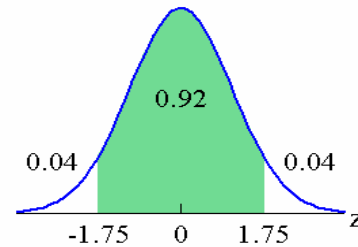
Area to the left of z_2 is 0.96

$$z_2 = 1.75$$

$$x_2 = m + z_2S$$

$$= 42.6 + 1.75 \cdot 6.5$$

$$= 53.98$$

**Question 3** (6 points)

The time it takes Mary to commute to school in the morning is normally distributed with a mean of 21 minutes and a standard deviation of 4.2 minutes.

- a) Find the probability that Mary will take more than 30 minutes to commute to school tomorrow morning.

$$\begin{aligned} P(x > 30) &= P(z > 2.14) \\ &= 1 - 0.9839 \\ &= 0.0162 \end{aligned}$$

$$z = \frac{x - m}{s} = \frac{30 - 21}{4.2} = 2.14$$

- b) Find the probability that Mary will take between 15 and 20 minutes to commute to school tomorrow morning.

$$\begin{aligned} P(15 < x < 20) &= P(-1.43 < z < -0.24) \\ &= 0.4052 - 0.0764 \\ &= 0.3288 \end{aligned}$$

$$z = \frac{x - m}{s} = \frac{15 - 21}{4.2} = -1.43$$

$$z = \frac{x - m}{s} = \frac{20 - 21}{4.2} = -0.24$$

- c) Find the 70th percentile.

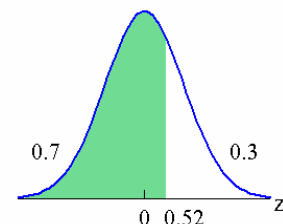
Area to the left of z is 0.70

$$z = 0.52$$

$$x = m + zS$$

$$= 21 + 0.52 \cdot 4.2$$

$$= 23.18 \text{ minutes}$$



Question 4 (4 points)

Express Courier Service has found that the delivery times for packages are normally distributed with mean 14 hours and standard deviation 2 hours.

- a) For a package selected at random, what is the probability that it will be delivered in 17 hours or less?

$$P(x < 17) = P(z < 1.50) \\ = 0.9332$$

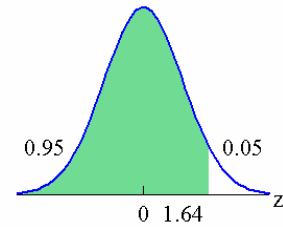
$$z = \frac{x - m}{s} = \frac{17 - 14}{2} = 1.50$$

- b) What should the guaranteed delivery time on all packages in order to be 95% sure that a given package will be delivered within this time?

Area to the left of z is 0.95

$$z = 1.645$$

$$x = m + z s \\ = 14 + 1.645 \cdot 2 \\ = 17.3 \text{ hours}$$



Question 5 (4 points) – *To be done with Excel*

The reaction time in a particular experiment is normally distributed with a mean of 0.7 seconds and a standard deviation of 0.12 seconds. Find the probability that an individual selected at random has a reaction time

- Greater than 0.9 seconds.
- Less than 0.6 seconds.
- Between 0.6 and 0.9 seconds.

Question 5

a)	$P(x > 0.9)$	0.0478
b)	$P(x < 0.6)$	0.2023
c)	$P(0.6 < x < 0.9)$	0.7499

Question 6 (11 points) – *To be done with Excel*

The mean IQ scores for students at a certain university is 105 with standard deviation 14.

- If a person is selected at random, what is the probability that his IQ is higher than 130?
- If a person is selected at random, what is the probability that his IQ is between 110 and 120.
- If a person is selected at random, what is the probability that his IQ is below 100.
- If the top 5% receive a bursary, what IQ score must be surpassed to receive it?
- Find the 10th percentile.
- Find the first quartile.
- Find the value of x such that only 20% of the IQ scores are below that value.
- Find the value of x such that only 15% of the IQ scores are above that value.
- Find the x values such that 99% of the area below the curve is between x_1 and x_2 .

Question 6

a)	$P(x > 130)$	0.0371
b)	$P(110 < x < 120)$	0.2185
c)	$P(x < 100)$	0.3605
d)	Area to the left = 0.95	128.03
e)	P10	87.06
f)	Q1	95.56
g)	Area to the left = 0.20	93.22
h)	Area to the left = 0.85	119.5
i)	Area to the left = 0.005	68.9
	Area to the left = 0.995	141.1