

## MATHEMATICS 201-NYC-05

Vectors and Matrices

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# XV - Spanning Sets and Linear Independence

- Determine if the vectors  $\vec{u}$ ,  $\vec{v}$  and  $\vec{w}$  can be written as linear combinations of the vectors in  $S = \{(2, -1, 3), (5, 0, 4)\}$ .
  - $\vec{u} = (0, -5, 7)$
  - $\vec{v} = (16, -\frac{1}{2}, \frac{27}{2})$
  - $\vec{w} = (3, 6, -2)$
- Determine if the vectors  $\vec{u}$ ,  $\vec{v}$  and  $\vec{w}$  can be written as linear combinations of the vectors in  $S = \{(2, 0, 7), (2, 4, 5), (2, -12, 13)\}$ .
  - $\vec{u} = (4, -20, 24)$
  - $\vec{v} = (-1, 0, 0)$
  - $\vec{w} = (6, 24, 9)$
- Determine whether the given set  $S$  spans  $\mathbb{R}^2$ . If the set does not span  $\mathbb{R}^2$ , give a geometrical description of the subspace that it does span.
  - $S = \{(2, 3), (-3, 2)\}$
  - $S = \{(-3, -4)\}$
  - $S = \{(3, -4), (-6, 8)\}$
- Determine whether the given set  $S$  spans  $\mathbb{R}^3$ . If the set does not span  $\mathbb{R}^3$ , give a geometrical description of the subspace that it does span.
  - $S = \{(2, -1, 3), (4, 1, -2), (8, -1, 4)\}$
  - $S = \{(1, -2, 3), (3, 4, -1)\}$
  - $S = \{(2, -1, 3), (4, -2, 6), (-6, 3, -9)\}$
  - $S = \{(1, 2, 3), (3, 2, 1), (1, 3, 1)\}$
- Determine whether the given set  $S$  spans  $P_2$ .
  - $S = \{5, x^2 + 5, 5x + 1\}$
  - $S = \{x - 1, x + 1, 10\}$
  - $S = \{x^2 - x + 1, 2x^2 + 5x - 3, -x^2 + 8x - 6\}$

6. Determine whether the set  $S$  is linearly independent or dependent.

a)  $S = \{(1, -3), (2, 4)\}$

b)  $S = \{(4, 1), (2, 2)\}$

c)  $S = \{(3, -3), (0, 0)\}$

d)  $S = \{(1, 1, 1), (-2, -2, -2), (5, 5, 5)\}$

e)  $S = \{(3, 2, 3), (4, -1, -2), (2, -1, 4), (4, 1, -5)\}$

f)  $S = \{(3, -1, 3), (1, 1, 1), (2, 2, 1)\}$

g)  $S = \{(0, 0, 0, 1), (0, 0, 1, 1), (0, 1, 1, 1), (1, 1, 1, 1)\}$

h)  $S = \{(1, -3, 2, 1), (-2, 5, 2, 1), (3, -8, 0, 0), (-1, 2, 4, 2)\}$

7. Show that the given set is linearly dependent by finding a nontrivial linear combination (of vectors in the set) whose sum is the zero vector. Then express one of the vectors in the set as a linear combination of the other vectors in the set.

a)  $S = \{(2, -3), (1, 4), (-2, 5)\}$

b)  $S = \{(1, -3, 2), (5, 2, 1), (3, 8, -3)\}$

8. For what values of  $t$  are the following sets linearly independent?

a)  $S = \{(t, 1, 1), (1, t, 1), (1, 1, t)\}$

b)  $S = \{(t, t, 1), (t, 0, t), (1, 1, t)\}$

9. Determine which of the sets in  $M_{2,2}$  are linearly independent.

a)  $S = \left\{ \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & -2 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 1 & -8 \\ 0 & 0 \end{bmatrix} \right\}$

b)  $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 4 \\ -3 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -8 \\ 0 & 0 \end{bmatrix} \right\}$

c)  $S = \left\{ \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}, \begin{bmatrix} 4 & 0 \\ -2 & 3 \end{bmatrix}, \begin{bmatrix} -1 & 2 \\ 0 & 6 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \right\}$

10. Determine which of the sets in  $P_2$  are linearly independent.

a)  $S = \{1 - x^2, 1 + x^2, 2x + 1\}$

b)  $S = \{x^2 + x + 1, x + 2, x^2 - 3x - 7\}$

c)  $S = \{x^2 + x - 1, x^2 - x + 1, x^2 + 2x + 1\}$

11. Use the Wronskian to determine if the subset  $W$  of  $C^2(-\infty, \infty)$  is linearly independent.

a)  $W = \{\sin x, \sin^2 x\}$

b)  $W = \{\sin x, \cos x, \sin^2 x\}$

c)  $W = \{1, \cos^2 x, \sin^2 x\}$

d)  $W = \{1, \sin x, \sin 2x\}$

e)  $W = \{e^x, xe^x, x^2e^x\}$

f)  $W = \{1-x, 1+4x, x^2, x^2+x\}$

## Answers

1. a)  $\vec{u} = 5(2, -1, 3) - 2(5, 0, 4)$       b)  $\vec{v} = \frac{1}{2}(2, -1, 3) + 3(5, 0, 4)$   
 c)  $\vec{w}$  cannot be written as a linear combination of the given vectors.
2. a)  $\vec{u} = 7(2, 0, 7) - 5(2, 4, 5) + 0(2, -12, 13)$   
 b)  $\vec{v}$  cannot be written as a linear combination of the given vectors  
 c)  $\vec{w} = -3(2, 0, 7) + 6(2, 4, 5) + 0(2, -12, 13)$ .
3. a)  $S$  spans  $\mathbb{R}^2$   
 b)  $S$  does not span  $\mathbb{R}^2$  (It spans the line  $4x - 3y = 0$ )  
 c)  $S$  does not span  $\mathbb{R}^2$  (It spans the line  $4x + 3y = 0$ )
4. a)  $S$  does not span  $\mathbb{R}^3$  (It spans the plane  $x - 16y - 6z = 0$ )  
 b)  $S$  does not span  $\mathbb{R}^3$  (It spans the plane  $x - y - z = 0$ )  
 c)  $S$  does not span  $\mathbb{R}^3$  (It spans the line  $\frac{x}{2} = \frac{y}{-1} = \frac{z}{3}$ )  
 d)  $S$  spans  $\mathbb{R}^3$
5. a)  $S$  spans  $P_2$   
 b)  $S$  does not span  $P_2$   
 c)  $S$  does not span  $P_2$
6. a) Linearly independent      b) Linearly independent  
 c) Linearly dependent      d) Linearly dependent  
 e) Linearly dependent      f) Linearly independent  
 g) Linearly independent      h) Linearly dependent
7. a)  $13(2, -3) - 4(1, 4) + 11(-2, 5) = (0, 0)$ ,       $(2, -3) = \frac{4}{13}(1, 4) - \frac{11}{13}(-2, 5)$   
 b)  $2(1, -3, 2) - (5, 2, 1) + (3, 8, -3) = (0, 0, 0)$ ,       $(1, -3, 2) = \frac{1}{2}(5, 2, 1) - \frac{1}{2}(3, 8, -3)$
8. a)  $t \neq -2, 1$       b)  $t \neq -1, 0, 1$
9. a) Linearly dependent      b) Linearly independent  
 c) Linearly independent
10. a) Linearly independent      b) Linearly dependent  
 c) Linearly independent
11. a) Linearly independent      b) Linearly independent  
 c) Linearly dependent      d) Linearly independent  
 e) Linearly independent      f) Linearly dependent