

MATHEMATICS 201-NYC-05

Vectors and Matrices

Martin Huard

Fall 2007

Vectors, Lines and Planes with Maple

When working with vectors on Maple, the first thing to do is to load the *LinearAlgebra* package, which contains a lot of the commands that we need.

```
> with(LinearAlgebra):
```

Vectors

Definition of Vectors with Maple

Vectors in Maple can be defined as points with the `Vector` command or using `< >`. For example, the vector $\vec{u} = (1, -2, 4)$ can be defined by

```
> u:=Vector([1,-2,4]);
```

or with

```
> u:=< 1 | -2 | 4 >;
```

Note that you can use the Vector palette on you left when entering them in this form. Also, you need to be consistent. Work with either one form or the other, but not a combination of both!

Plotting Vectors

We can represent vectors graphically using the `arrow` and `display` commands from the *plots* package.

```
> with(plots):
```

The `arrow` command has the following format: `arrow(vector , options)` or `arrow(base, direction , options)` if we don't want the vector to start at the origin. For the options, you may have to use `width = ...` if the arrow appears too small.

For example, let us plot the vector $\vec{u} = (1, -2, 4)$.

```
> gu:=arrow(u);  
display(gu, axes=normal, labels=[x,y,z]);
```

The angle of vision can be moved by clicking on the graph and moving your mouse (with the button still pressed down). Try it!

Operations with vectors

Maple can handle most of the common operations with vectors. For example, suppose we have the points $A(2, -3, 1)$, $B(2, -1, 5)$ and $C(-1, -3, 4)$, which are entered as vectors,

```
> A:=< 2 | -3 | 1 >;
   B:=< 2 | -1 | 5 >;
   C:=< -1 | -3 | 4 >;
```

Then the vectors \overrightarrow{AB} and \overrightarrow{AC} are given by

```
> AB:=B-A;
   AC:=C-A;
```

Vector Addition

The addition of vectors is done as expected. For example, $\overrightarrow{AB} + \overrightarrow{AC}$ is given by

```
> AB+AC;
```

Multiplication by a scalar

The vector $4\overrightarrow{AB}$ is given by

```
> 4*AB;
```

Magnitude of a vector

Maple has the command **VectorNorm** in the *LinearAlgebra* package to find the magnitude of a vector. For example, let us find $\|\overrightarrow{AB}\|$

```
> VectorNorm(A, Euclidean);
```

Thus, $\|\overrightarrow{AB}\| = 2\sqrt{5}$.

Scalar or dot product

The command for the scalar product of two vectors is **DotProduct** taken from the *LinearAlgebra* package.

```
> DotProduct(AB, AC);
```

Hence, $\overrightarrow{AB} \cdot \overrightarrow{AC} = (0, 2, 4) \cdot (-3, 0, 3) = 12$.

Vector or cross product

For the vector product, we use the **CrossProduct** command from the *LinearAlgebra* package.

```
> CrossProduct(AB, AC);
```

Hence $\overrightarrow{AB} \times \overrightarrow{AC} = (0, 2, 4) \times (-3, 0, 3) = (6, -12, 6)$

Angle between two vectors

We can find the angle between two vectors using the angle command. For example, let us find the angle between the vectors \overline{AB} and \overline{AC} .

```
> theta:=VectorAngle(AB,AC);
```

To obtain a decimal answer, we can use the **evalf** command.

```
> evalf(theta);
```

Note that Maple gives the answer in radians. To have the angle in degrees, we use the **convert** command.

```
> convert(theta,degrees);
evalf(%);
```

Hence $\theta \approx 50.8^\circ$.

Lines

We can plot lines in \mathbb{R}^3 with Maple when the line is given in vector form. For example, if we have the line passing through the points A and B , $l : (x, y, z) = (1, 3, -5) + t(2, -3, 4)$, then we can define it

```
> l:=A+t*AB;
```

and use the command **spacecurve** from the *plots* package to graph it.

```
> with(plots):
```

```
gline:=spacecurve(evalm(l),t=-3..3):
```

```
display(gline, axes=normal, labels=[x,y,z],scaling=constrained);
```

Planes

Vector Form

To plot a plane in vector form, such as the plane passing through the points A , B and C , given by $\pi : (x, y, z) = (2, -3, 1) + s(0, 2, 4) + t(-3, 0, 3)$ we start by defining it,

```
> pi:= A+s*AB+t*AC;
```

and then use the **plot3d** command.

```
> gplane:=plot3d(evalm(pi),t=-2..2,s=-2..2):
```

```
display(gplane, axes=normal, labels=[x,y,z]);
```

General Form

To plot a plane in general form, such as $\pi_2 : 5x - 2y - z = 5$, we have

```
> pi2:=5*x-2*y-z=5;
```

and use the **implicitplot3d** command.

```
> gplane2:=implicitplot3d(pi2,x=-8..8,y=-8..8,z=-8..8):
```

```
> display(gplane2, axes=normal, labels=[x,y,z]);
```