

MATHEMATICS 201-NYC-05

Vectors and Matrices

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Definition of a Vector Space

Let V be a set on which two operations (*vector addition* \oplus and *scalar multiplication* \odot) are defined. If the following axioms are satisfied for every \vec{u} , \vec{v} and \vec{w} in V and every scalar k and l , then V is called a *vector space*.

Vector Addition

1. $\vec{u} \oplus \vec{v} \in V$ closure under addition
2. $\vec{u} \oplus \vec{v} = \vec{v} \oplus \vec{u}$ commutativity
3. $\vec{u} \oplus (\vec{v} \oplus \vec{w}) = (\vec{u} \oplus \vec{v}) \oplus \vec{w}$ associativity
4. V has a *zero vector* $\vec{0}$ such that for every \vec{u} in V , $\vec{u} \oplus \vec{0} = \vec{u}$ additive identity
5. For every \vec{u} in V , there is a vector in V , denoted $-\vec{u}$,
such that $\vec{u} \oplus (-\vec{u}) = \vec{0}$ additive inverse

Scalar Multiplication

6. $k \odot \vec{u} \in V$ closure under scalar multiplication
7. $(k \oplus l) \odot \vec{u} = (k \odot \vec{u}) \oplus (l \odot \vec{u})$ distributivity
8. $k \odot (\vec{u} \oplus \vec{v}) = (k \odot \vec{u}) \oplus (k \odot \vec{v})$ distributivity
9. $k \odot (l \odot \vec{u}) = (kl) \odot \vec{u}$ associativity
10. $1 \odot \vec{u} = \vec{u}$ scalar identity

Test for a Subspace

If W is a nonempty subset of a vector space V , then W is a subspace of V if and only if

1. If \vec{u} and \vec{v} are in W , then $\vec{u} \oplus \vec{v} \in W$
2. If \vec{u} is in W and k is a scalar, then $k \odot \vec{u} \in W$