

MATHEMATICS 201-NYC-05

Vectors and Matrices

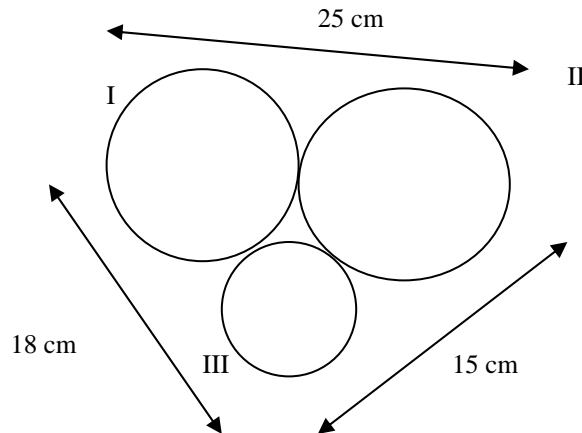
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VI - Applications

1. The perimeter of a triangle is 72 cm. The longest side is 10 cm more than twice the shortest side, and twice the middle side is 2 cm more than the longest side. Find the length of the sides of a triangle.
2. A small school has 100 students who occupy three classrooms: rooms A, B, C. After the first period of the school day, half the students in room A move to room B, one-fifth of the students in room B move to room C, and one-third of the students in room C move to room A. Nevertheless, the total number of students in each room is the same for each period. How many students occupy each room?
3. A father and his three daughters are celebrating their birthday today. Knowing that the father's age is twice the sum of the age of his daughters, and thirteen times the difference in age between the two youngest daughters, and also knowing that the age of the eldest is three times the difference in age of her sisters, and that a quarter of the age of the eldest is equal to a third the age of the second oldest, determine the age of each person on this anniversary day.
4. A chemist has three containers of acid solutions at various concentrations. The first is 10% acid, the second is 20%, and the third is 40%. How many milliliters of each should he mix together to make 100 mL of acid at 18% concentration, if he has to use four times as much of the 10% solutions as the 40% solution?
5. A glass of skim milk supplies 0.1 mg of iron, 8.5 g of protein and 1 g of carbohydrates. A quarter pound of lean red meat provides 3.4 mg of iron, 22 g of protein, and 20 g of carbohydrates. Two slices of whole-grain bread supply 2.2 mg of iron, 10 g of protein, and 12 g of carbohydrates. If a person on a special diet must have 12.1 mg of iron, 97 g of protein, and 70 g of carbohydrates, how many glasses of skim milk, how many quarter-pound servings of meat, and how many two-slice servings of whole-grain bread will supply this?
6. Three different bacteria are cultured in one dish and feed on three nutrients. Each individual of species I consumes one unit of each of the first and second nutrients and two units of the third nutrient. Each individual of species II consumes two units of the first nutrient and two units of the third nutrient. Species III consumes two units of the first nutrient, three units of the second nutrient, and five units of the third nutrient. If the culture is given 5100 units of the first nutrient, 6900 units of the second nutrient, and 12000 units of the third nutrient, how many of each species can be supported so all of the nutrients are consumed?

7. Find the radius of each cylinder.



8. Gauss is either happy or sad. If he is happy one day, he is happy the next day four times out of five. If he is sad one day, he is sad the next day one time out of three.

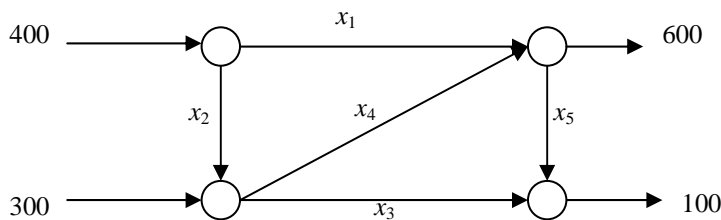
- a) If Gauss is sad today, what is the probability he will be sad two days from now?
- b) Over the long term, what are the chances that John is happy on any given day?

9. A country is divided into three demographic regions. It is found that each year 5% of the residents of region 1 move to region 2, and 5% move to region 3. Of the residents of region 2, 15% move to region 1 and 10% move to region 3. And of the residents of region 3, 10% move to region 1 and 5% move to region 2. Currently, the population is equally divided into the three regions.

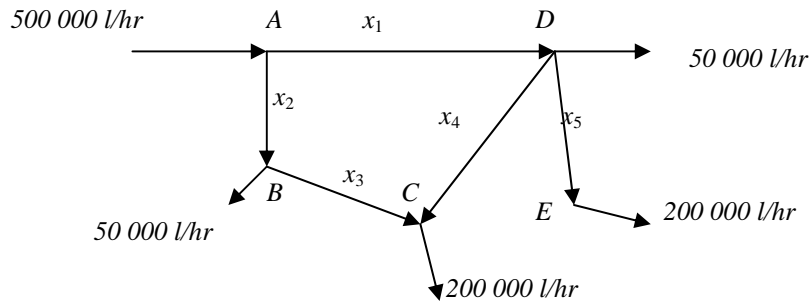
- a) Find the transition matrix.
- b) What percentage of the population resides in each of the three regions after a two years?
- c) What percentage of the population resides in each of the three regions after a long period of time?

10. Water is flowing though a network of pipes (in thousands of cubic meters per hour), as show below.

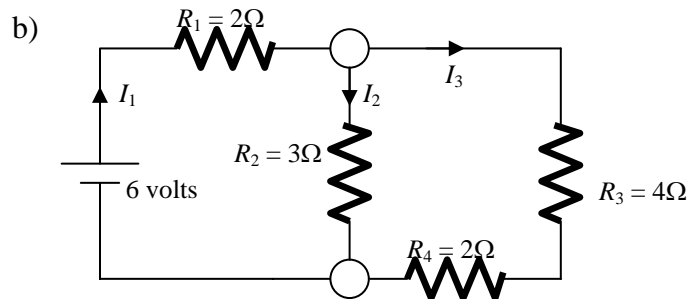
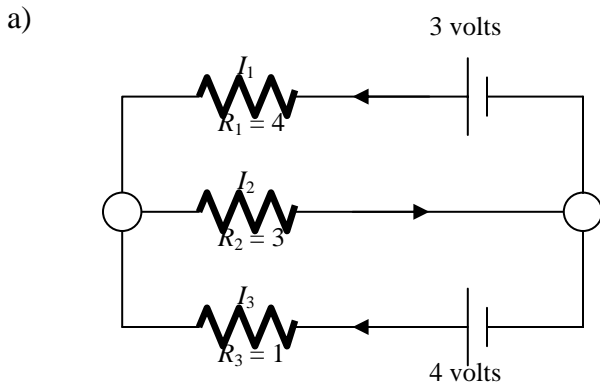
- a) Solve this system for the water flow.
- b) Find the network flow when $x_1 = 200$, $x_2 = 200$.
- c) Find the network flow when $x_5 = 200$.



11. An irrigation system allows water to flow in the pattern shown below. Water flows into the system at A and exits at B , C , D and E with the amounts shown. Find the flow of water at each of the five points.



12. Determine the currents I_1 , I_2 and I_3 for each of the electrical networks.



13. Find the equation of the parabola (if possible) passing through the given points.

- a) $(1,0)$, $(2,3)$, $(-4,45)$
- b) $(2,-1)$, $(-3,-6)$, $(4,-13)$
- c) $(-3,4)$, $(2,1)$, $(-3,5)$
- d) $(-2,-7)$, $(2,1)$, $(3,3)$

14. Find the equation of the third degree polynomial $y = ax^3 + bx^2 + cx + d$ passing through the points $(1,4)$, $(2,21)$, $(-1,-6)$ and $(-2,-23)$.

Answers

1. 38cm, 20cm and 14 cm
2. 20 in room A, 50 in room B and 30 in room C
3. Father : 52 and the daughters 12, 9 and 5.
4. 40 ml of the 10% solution, 50 ml of the 20% and 10 ml of the 40%
5. $\frac{3}{8}$ lb of red meat, 6 slices of bread and 4 glasses of milk
6. Bacteria III : any amount t (between 1800 and 2300)
 Bacteria II = $\frac{1}{2}t - 900$
 Bacteria I = $6900 - 3t$
7. I – 7cm, II – $\frac{11}{2}$ cm and III – 2 cm
8. a) $\frac{58}{75} \approx 77.3\%$ b) $\frac{10}{13}$
9. a) $\begin{bmatrix} 0.90 & 0.15 & 0.10 \\ 0.05 & 0.75 & 0.05 \\ 0.05 & 0.10 & 0.85 \end{bmatrix}$
- b) $(\frac{101}{240}, \frac{149}{600}, \frac{397}{1200}) \approx (42.1\%, 24.8\%, 33.1\%)$
 41.1% in region 1, 24.8% in region 2 and 33.1% in region 3
- c) $(\frac{13}{24}, \frac{1}{6}, \frac{7}{24}) \approx (54.2\%, 16.7\%, 29.2\%)$
 54.2% in region 1, 16.7% in region 2 and 29.2% in region 3
10. a) $x_1 = 600 + t - s$, $x_2 = -200 - t + s$, $x_3 = 100 - t$, $x_4 = s$ and $x_5 = t$
 b) $x_1 = 200$, $x_2 = 200$, $x_3 = 100 - t$, $x_4 = 400 + t$ and $x_5 = t$
 c) impossible
11. $x_1 = 250\,000 + t$, $x_2 = 250\,000 - t$, $x_3 = 200\,000 - t$, $x_4 = t$ and $x_5 = 200\,000$
12. a) $I_1 = 0$ A, $I_2 = 1$ A, $I_3 = 1$ A. b) $I_1 = \frac{3}{2}$ A, $I_2 = 1$ A, $I_3 = \frac{1}{2}$ A.
13. a) $2x^2 - 3x + 1$ b) $-x^2 + 3$ c) No solution
 d) No parabola but the line $y = 2x - 3$
14. $2x^3 + 3x - 1$