MATHEMATICS 201-NYC-05
Vectors and Matrices
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Assignment #4

This assignment is due ***Friday November 23*** at the beginning of the class. Complete solutions are expected.

Question 1 (12 points)

Prove that $W=\{(s,5,t): s,t\in\mathbb{R}\}$ with the following definitions for vector sum and scalar multiplication is a vector space. (Verify all 10 axioms!)

$$(u_1,5,u_3)\oplus(v_1,5,v_3)=(u_1+v_1,5,u_3+v_3+1)$$

$$k\odot(u_1,5,u_3)=(ku_1,5,k+ku_3-1)$$

Question 2 (10 points)

For what values of *t* is the set $S=\{tx^2+x+2, x^2+tx+2, tx^2+3x+t\}$ linearly independent?

Question 3 (20 points)

Let $W=\{p(x): p(x)\in P_3,\ p(3)=0\}$ and $U=\{p(x): p(x)\in P_3,\ p(2)=0,\ p(-2)=0\}$.

a) Show that *W* and *U* are subspaces of P_3.
b) Find a basis and the dimension for *W*.
c) Find a basis and the dimension for *U*.
d) Is $q(x)=2x^3-6x^2-8x+24$ an element of W? If yes, find the coordinates of $q(x)$ relative to the basis found in (b).
e) Find a basis for $W\cap U$ and determine the dimension of $W\cap U$.
f) Is $q(x)=2x^3-6x^2-8x+24$ an element of $W\cap U$? If yes, find the coordinates of $q(x)$ relative to the basis found in (e).

Question 4 (8 points)

Let $\{\vec{v}_1,\vec{v}_2,\vec{v}_3\}$ be an orthonormal basis for $\mathbb{R}^3$.

a) Show that if $\vec{w}\in\mathbb{R}^3$, then $\vec{w}$ can be written as $\vec{w}=(\vec{w}\bullet\vec{v}_1)\vec{v}_1+(\vec{w}\bullet\vec{v}_2)\vec{v}_2+(\vec{w}\bullet\vec{v}_3)\vec{v}_3$.
b) Show that if $\vec{w}\in\mathbb{R}^3$, then $\|\vec{w}\|^2=(\vec{w}\bullet\vec{v}_1)^2+(\vec{w}\bullet\vec{v}_2)^2+(\vec{w}\bullet\vec{v}_3)^2$.