**MATHEMATICS 201-NYC-05**
Vectors and Matrices
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# Assignment #3

This assignment is due ***Friday November 2*** at the beginning of the class.
For questions involving Maple, a print-out of your work is expected, where your name is written in the Worksheet, and each question is clearly labeled. To write text in Maple, you can click on T or go to INSERT – TEXT.

## Question 1 (7 points)

Let $\vec{u}$ and $\vec{v}$ be nonzero vectors. Let $\vec{w} = k\vec{u} + l\vec{v}$ where $k = \|\vec{v}\|$ and $l = \|\vec{u}\|$. Show that $\vec{w}$ bisects the angle between $\vec{u}$ and $\vec{v}$. That is, if $\theta$ is the angle between $\vec{u}$ and $\vec{v}$, then the angle between $\vec{u}$ and $\vec{w}$ is $\frac{\theta}{2}$, and the angle between $\vec{v}$ and $\vec{w}$ is $\frac{\theta}{2}$. *Hint*: Use the trigonometric identity $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$.

## Question 2 (20 points)

For a given set of vectors $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ in $\mathbb{R}^3$. We define the vectors $\vec{w}_1$, $\vec{w}_2$ and $\vec{w}_3$ as

$$\vec{w}_1 = \vec{v}_1$$

$$\vec{w}_2 = \vec{v}_2 - \frac{\vec{v}_2 \bullet \vec{w}_1}{\vec{w}_1 \bullet \vec{w}_1}\vec{w}_1$$

$$\vec{w}_3 = \vec{v}_3 - \frac{\vec{v}_3 \bullet \vec{w}_1}{\vec{w}_1 \bullet \vec{w}_1}\vec{w}_1 - \frac{\vec{v}_3 \bullet \vec{w}_2}{\vec{w}_2 \bullet \vec{w}_2}\vec{w}_2$$

and the vectors $\vec{u}_1$, $\vec{u}_2$ and $\vec{u}_3$ by

$$\vec{u}_1 = \frac{\vec{w}_1}{\|\vec{w}_1\|} \qquad \vec{u}_2 = \frac{\vec{w}_2}{\|\vec{w}_2\|} \qquad \vec{u}_3 = \frac{\vec{w}_3}{\|\vec{w}_3\|}$$

This is known as the ***Gram-Schmidt Orthonormalization Process***.

a) For the vectors $\vec{v}_1 = (1,0,1)$, $\vec{v}_2 = (1,1,2)$ and $\vec{v}_3 = (1,2,1)$, find $\vec{w}_1$, $\vec{w}_2$, $\vec{w}_3$, $\vec{u}_1$, $\vec{u}_2$ and $\vec{u}_3$.
b) Show that the set $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ found in (a) is orthonormal.
c) Prove that for any nonzero vectors $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$ such that $\vec{w}_1$, $\vec{w}_2$ and $\vec{w}_3$ are all nonzero (this is equivalent to saying that the vectors $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$ are linearly independent), then
   i) $\{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$ is an orthogonal set
   ii) $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ is an orthonormal set

d) Verify your answer in (a) with Maple. That is, find the vectors $\vec{w}_1$, $\vec{w}_2$, $\vec{w}_3$, $\vec{u}_1$, $\vec{u}_2$ and $\vec{u}_3$ with Maple.

e) Let $Q$ be the matrix in which has for rows the vector $\vec{u}_1$, $\vec{u}_2$ and $\vec{u}_3$. Find the inverse of $Q$ with Maple. What is the relationship between $Q$ and its inverse?

# Question 3 (7 points)

Consider the points $A(2,-1,3)$, $B(6,3,-3)$ and $C(6,-7,-4)$. Using Maple,

a) find the equation of the line *l* passing through the points *A* and *B*;
b) plot the line *l* found in (a) along with the direction vector for the line (choose a view such that both objects, and the relationship between them are clearly seen);
c) find the distance between the point *C* and the line *l* found in (a);
d) find the point *Q* on *l* closest to the point *C*.
e) find the distance between the line *l* found in (a) and the line $l_2 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-1}{4}$.

# Question 4 (9 points)

Consider the four points $A(2,-1,3)$, $B(3,1,1)$, $C(-2,-2,1)$ and $P(14,-6,-6)$. Using Maple,

a) find the equation of the plane π passing through the points *A*, *B* and *C* (in general form);
b) find the equation of the line *l* passing through *P* and perpendicular to the plane π;
c) find the point *Q* on π closest to the point *P*;
d) find the distance between the point *P* and the plane π;
e) plot the line *l*, the plane π and the normal vector (choose a view such that all three objects, and the relationships between them, are clearly seen);
f) find the volume of the tetrahedron *ABCP*.

# Question 5 (7 points)

Consider the planes $\pi_1 : x-3y+z=8$ and $\pi_2 : 5x-2y+3z=5$. Using Maple,

a) find the intersection of the planes $\pi_1$ and $\pi_2$ (Expressed in vector form) ;
b) find the angle between the two planes (answer should be in degrees);
c) find the equation for a plane $\pi_3$ that is not parallel to either $\pi_1$ and $\pi_2$ such that $\pi_1 \cap \pi_2 \cap \pi_3 = \varnothing$; (Verify your answer!)
d) plot the three planes (choose a view that shows why the planes do not all intersect at a point).