MATHEMATICS 201-NYC-05
Vectors and Matrices
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Fall 2007

Assignment #2

This assignment is due ***Monday October 1*** at the beginning of the class.
For questions involving Maple, a print-out of your work is expected, where your name is written in the Worksheet, and each question is clearly labeled.

Question 1 (26 points)

Consider the matrix $A=\begin{bmatrix} 2 & 2 & 0 \\ -1 & 2 & 3 \\ 1 & 2 & -1 \end{bmatrix}$.

a) Find the eigenvalues and eigenvectors of A.
b) Verify that $\operatorname{tr}(A)=\lambda_1+\lambda_2+\lambda_3$.
c) Verify that $\det(A)=\lambda_1\lambda_2\lambda_3$.
d) Consider the matrix P whose columns are the eigenvectors of A found in (a). Find P.
e) Find P^{-1}.
f) Use (d) and (e) to find the matrix D defined as $D=P^{-1}AP$. What do the entries in D correspond to? (What was done in (a), (d), (e) and (f) is referred to as the diagonalization of A)
g) Show that if P is an invertible matrix such that $D=P^{-1}AP$, then $A=PDP^{-1}$.
h) If P is an invertible matrix such that $A=PDP^{-1}$ and k is a positive integer, then $A^k=PD^kP^{-1}$. Prove this result for $k=3$, that is, prove that $A^3=PD^3P^{-1}$.
i) Use the result from (h) to find A^5 (This is a procedure that enables a relatively quick calculation of powers of matrices when the exponent is large).
j) Verify your answers in (a), (b), (c), (e), (f) and (i) using Maple.

Question 2 (4 points)

Use Maple to find the lowest degree polynomial (if any) passing through the points $(-2,67)$, $(-1,4)$, $(1,-2)$, $(3,112)$ and $(5,970)$. Your solution should be clear, that is, write in Text in the Maple worksheet what you are doing or finding.

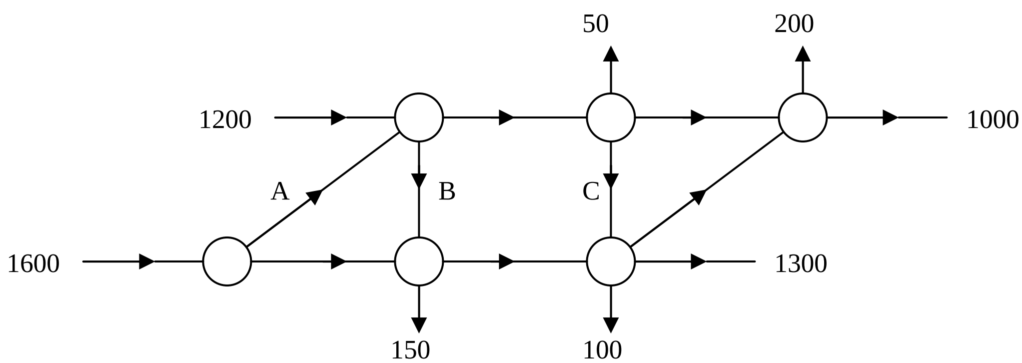
Question 3 (10 points)

John Doe is an avid reader of science fiction, math books and philosophy. He reads one book every week. If he is presently reading science fiction, there is a 60% chance he will switch to a math book next week, and a 40% chance he will switch to philosophy. If he is presently reading a math book, there is a 50% chance that he will read another one the next week, with a 25% percent chance that he will switch to science fiction. If he is presently reading philosophy, there is a 75% chance that he will switch to science fiction the next week, and he never reads two philosophy books in a row.

a) Find the transition matrix.
b) If John is currently reading a math book, what is the probability he will also be reading a math book two weeks from now?
c) If he is currently reading a math book, what is the probability he will also be reading a math book *m* weeks from now, where *m* is the number corresponding to the last 3 digits of your student number? Use Maple, where the answer should be given as a percentage, with three significant digits.
d) Assuming this trend continues indefinitely, what fraction of the book he reads are math ones?
e) Find the eigenvalues of the transition matrix using Maple.
f) Using Maple, find an eigenvector, whose sum of the entries is one, associated with the biggest eigenvalue found in (d). Compare your answer to what you obtained in (c).

Question 4 (5 points)

Water is flowing through a network of pipes (in cubic meters per hour) as shown below.



a) Set up the system of linear equations (without evaluating it).
b) Solve the system of linear equations using Maple.
c) What should the flow be if pipes A, B and C are broken and the flow through them is zero?

Question 5 (5 points)

Let *A*, *B* and *C* be three points, not in a straight line such that $\overrightarrow{AD} = \frac{2}{7}\overrightarrow{AB} + \frac{5}{7}\overrightarrow{AC}$.

a) Prove that $\overrightarrow{BC}$ and $\overrightarrow{CD}$ are parallel vectors.
b) If $\left\|\overrightarrow{BC}\right\| = 21$, find $\left\|\overrightarrow{CD}\right\|$.