**MATHEMATICS 201-NYB-05**
Integral Calculus
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Winter 2012

# VII - Trigonometric Integrals

1. Evaluate the integral.

   a) $\int \cos^5 x \sin x \, dx$ b) $\int \cos^3 x \sin^6 x \, dx$ c) $\int 3\sin^2(7x)\, dx$

   d) $\int \cos^5 \theta \, d\theta$ e) $\int \sin^2(5t)\cos^2(5t)\, dt$ f) $\int \frac{\sin^{\frac{3}{2}} x \cos^2 x + \sin x \cos x}{\sqrt{\sin x}} dx$

   g) $\int \frac{\sin^5 x - \sin^3 x}{1-\cos x} dx$ h) $\int \cos^4 x \, dx$ i) $\int \frac{2}{\cos^2(3x-1)} dx$

   j) $\int \tan^{\frac{2}{3}} x \sec^2 x \; dx$ k) $\int \tan^3 t \sec^3 t \; dt$ l) $\int \sec^5 x \tan^3 x \, dx$

   m) $\int \tan^2 x \sec x \; dx$ n) $\int \sec^4 x \, dx$ o) $\int \sec^5 x \, dx$

   p) $\int \tan^3 x \, dx$ q) $\int \tan^4 x \, dx$ r) $\int \tan^5(2x)\, dx$

   s) $\int \cot^2 \theta \csc^4 \theta \, dx$ t) $\int \csc 3x \, dx$ u) $\int \csc^3 x \, dx$

   v) $\int \sin 3x \cos 5x \, dx$ w) $\int \cos 7\theta \cos 5\theta \, d\theta$ x) $\int \sin 6x \sin 3x \, dx$

2. Evaluate the given integral.

   a) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos^3 x}{\sqrt{\sin x}} \; dx$ b) $\int_0^{\frac{\pi}{6}} \sin^2 3x \cos^5 3x \, dx$ c) $\int_0^{\frac{\pi}{2}} \sin^4 x \, dx$

   d) $\int_{-\frac{\pi}{2}}^{0} \cos^3 x \sin x \, dx$ e) $\int_{\frac{\pi}{9}}^{\frac{\pi}{6}} \cot 3x \, dx$ f) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^4 x \, dx$

   g) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^5 x \, dx$ h) $\int_{-\pi}^{\pi} \sin 3\theta \cos \theta \, d\theta$ i) $\int_0^{\frac{\pi}{4}} \sin 5\theta \sin \theta \, d\theta$

3. Prove the following reduction formula for powers of secant.

   $$\int \sec^n x \, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$$

4. Use the reduction formula from question 3 to evaluate $\int \sec^5 x \, dx$.

5. Let $m$ and $n$ be distinct nonnegative integers. Prove the following.

   a) $\int_0^{2\pi} \sin mx \cos nx \, dx = 0$

   b) $\int_0^{2\pi} \sin mx \sin nx \, dx = 0$

   c) $\int_0^{2\pi} \cos mx \cos nx \, dx = 0$

# Answers

1. a) $-\frac{1}{6}\cos^6 x + C$ or $\frac{1}{2}\sin^2 x - \frac{1}{2}\sin^4 x + \frac{1}{6}\sin^6 x + C$
   b) $\frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + C$
   c) $\frac{3}{2}x - \frac{3}{28}\sin(14x) + C$
   d) $\sin\theta - \frac{2}{3}\sin^3\theta + \frac{1}{5}\sin^5\theta + C$
   e) $\frac{1}{8}x - \frac{1}{160}\sin(20x) + C$
   f) $\frac{2}{3}\sin^{\frac{3}{2}} x - \frac{1}{3}\cos^3 x + C$
   g) $\frac{1}{4}\cos^4 x + \frac{1}{3}\cos^3 x + C$
   h) $\frac{3}{8}x + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C$
   i) $\frac{2}{3}\tan(3x-1) + C$
   j) $\frac{5}{3}\tan^{\frac{3}{5}} x + C$
   k) $\frac{1}{5}\sec^5 t - \frac{1}{3}\sec^3 t + C$
   l) $\frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$
   m) $\frac{1}{2}\sec x\tan x - \frac{1}{2}\ln|\sec x + \tan x| + C$
   n) $\tan x + \frac{1}{3}\tan^3 x + C$
   o) $\frac{1}{4}\sec^3 x\tan x + \frac{3}{8}\sec x\tan x + \frac{3}{8}\ln|\sec x + \tan x| + C$
   p) $\frac{1}{2}\tan^2 x + \ln|\cos x| + C$ or $\frac{1}{2}\tan^2 x - \ln|\sec x| + C$
   q) $\frac{1}{3}\tan^3 x - \tan x + x + C$
   r) $\frac{1}{8}\tan^4(2x) - \frac{1}{4}\tan^2(2x) + \frac{1}{2}\ln|\sec(2x)| + C$
   s) $-\frac{1}{5}\cot^5\theta - \frac{1}{3}\cot^3\theta + C$
   t) $\frac{1}{3}\ln|\csc 3x - \cot 3x| + C$
   u) $-\frac{1}{2}\csc x\cot x + \frac{1}{2}\ln|\csc x - \cot x| + C$
   v) $-\frac{1}{16}\cos 8x + \frac{1}{4}\cos 2x + C$
   w) $\frac{1}{4}\sin 2\theta + \frac{1}{24}\sin 12\theta + C$
   x) $\frac{1}{6}\sin 3x - \frac{1}{18}\sin 9x + C$
2. a) $\frac{8}{5} - \frac{19\sqrt{2}}{20}$ b) $\frac{8}{315}$ c) $\frac{3\pi}{16}$
   d) $-\frac{1}{4}$ e) $\frac{1}{3}\ln 2 - \frac{1}{6}\ln 3$ f) $\frac{\pi}{4} - \frac{2}{3}$
   g) $\frac{1}{2}\ln 2 - \frac{1}{4}$ h) $0$ i) $\frac{1}{12}$
4. $\frac{1}{4}\sec^3 x\tan x + \frac{3}{8}\sec x\tan x + \frac{3}{8}\ln|\sec x + \tan x| + C$