CEGEP CHAMPLAIN ST. LAWRENCE
CHAMPLAIN REGIONAL COLLEGE

MATHEMATICS 201-NYB-05
Integral Calculus
Martin Huard
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V – Substitution (*u*- *Subs*)

1. Evaluate the indefinite integral.

a) $\int x\left(x^2+1\right)^{23}\,dx$ b) $\int \dfrac{x}{\left(4x^2+9\right)^2}\,dx$ c) $\int \dfrac{3}{\sqrt{4x+1}}\,dx$

d) $\int x\sqrt{x^2-1}\,dx$ e) $\int \dfrac{1}{\sqrt{x}}\sin\sqrt{x}\,dx$ f) $\int \sec^2(3x+5)\,dx$

g) $\int e^{2x}\sqrt{1+e^{2x}}\,dx$ h) $\int \dfrac{\sin 7x}{3+\cos 7x}\,dx$ i) $\int \cos x\, e^{\sin x}\,dx$

j) $\int \dfrac{\sin\left(\frac{3}{x}\right)}{x^2}\,dx$ k) $\int \dfrac{1}{x\ln x}\,dx$ l) $\int \sec^3 x\tan x\,dx$

m) $\int \dfrac{1+x}{1+x^2}\,dx$ n) $\int \dfrac{(\ln x)^2}{x}\,dx$ o) $\int \sqrt{e^x}\,dx$

p) $\int \dfrac{4x+6}{\sqrt{x^2+3x+1}}\,dx$ q) $\int x\sin^5\left(x^2\right)\cos\left(x^2\right)dx$ r) $\int \dfrac{\sec^2 x}{\sqrt{1+\tan x}}\,dx$

2. Evaluate the definite integral.

a) $\int_0^2 (x+1)^5\,dx$ b) $\int_0^1 e^{2x-1}\,dx$ c) $\int_0^4 \dfrac{1}{3x+4}\,dx$

d) $\int_1^3 \dfrac{x}{\sqrt{x^2+16}}\,dx$ e) $\int_0^{\frac{\pi}{4}} \sin 4x\,dx$ f) $\int_3^6 \dfrac{1}{(x-2)^3}\,dx$

g) $\int_0^1 xe^{-x^2}\,dx$ h) $\int_1^3 \dfrac{1}{x^2}\sqrt{1+\dfrac{1}{x}}\,dx$ i) $\int_0^{13} \dfrac{1}{\sqrt[3]{(1+2x)^2}}\,dx$

j) $\int_0^e \dfrac{1}{x+e}\,dx$ k) $\int_1^{\sqrt{2}} \dfrac{x}{1+x^4}\,dx$ l) $\int_0^{\sqrt{2}} \dfrac{4}{2+x^2}\,dx$

m) $\int_1^8 x\sqrt{3x+1}\,dx$ n) $\int_4^{16} \dfrac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}\,dx$ o) $\int_{-1}^{11} \dfrac{3x-1}{\sqrt{2x+3}}\,dx$

3. Evaluate the following definite integrals.

a) $\int_0^a x\sqrt{x^2+a^2}\,dx$ b) $\int_0^1 x(1-x)^n\,dx$ where n is a positive integer

4. Evaluate the indefinite integrals.

a) $\int \dfrac{ax+b}{\sqrt{ax^2+2bx+c}}\,dx$ b) $\int \dfrac{1}{a^2+b^2x^2}\,dx$ c) $\int \cot x\,dx$

5. If f is continuous and $\int_0^{12} f(x)\,dx = 21$, find $\int_0^4 f(3x)\,dx$.

6. The rate of infection of a disease (in people per month) is given by $I'(t) = \dfrac{100t}{t^2+1}$ where t is the time in months since the disease broke out. Find the total number of infected people over the first six months of the disease.

7. The rate of air flow (in liters per second) into the lungs can be modeled by $A'(t) = \frac{1}{2}\sin\left(\frac{2\pi t}{7}\right)$. Find the total change in volume during the first 3.5 seconds and the first 7 seconds.

8. Find the average value of the function on the given interval.

   a) $f(x) = x^4\sqrt{2x^5-2}$ on $[1,3]$

   b) $f(x) = \dfrac{\sin 3x}{(2-\cos 3x)^3}$ on $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$

Answers

1. a) $\frac{1}{48}\left(x^2+1\right)^{24}+C$ b) $\dfrac{-1}{8\left(4x^2+9\right)}+C$ c) $\frac{3}{2}\sqrt{4x+1}+C$

   d) $\frac{1}{3}\left(x^2-1\right)^{\frac{3}{2}}+C$ e) $-2\cos\sqrt{x}+C$ f) $\frac{1}{3}\tan(3x+5)+C$

   g) $\frac{1}{3}\left(1+e^{2x}\right)^{\frac{3}{2}}+C$ h) $-\frac{1}{7}\ln(3+\cos 7x)+C$ i) $e^{\sin x}+C$

   j) $\frac{1}{3}\cos\left(\frac{3}{x}\right)+C$ k) $\ln\left|\ln|x|\right|+C$ l) $\frac{1}{3}\sec^3 x+C$

   m) $\arctan x+\frac{1}{2}\ln\left(1+x^2\right)+C$ n) $\frac{1}{3}(\ln x)^3+C$ o) $2e^{\frac{1}{2}x}+C$

   p) $4\sqrt{x^2+3x+1}+C$ q) $\frac{1}{12}\sin^6\left(x^2\right)+C$ r) $2\sqrt{1+\tan x}+C$

2. a) $\frac{364}{3}$ b) $\frac{e}{2}-\frac{1}{2e}$ c) $\frac{2}{3}\ln 2$ d) $5-\sqrt{17}$ e) $\frac{1}{2}$

   f) $\frac{15}{32}$ g) $\frac{1}{2}-\frac{1}{2e}$ h) $\frac{4}{3}\sqrt{2}-\frac{16}{27}\sqrt{3}$ i) 3 j) $\ln 2$

   k) $\frac{1}{2}\arctan 2-\frac{\pi}{8}$ l) $\frac{\pi\sqrt{2}}{2}$ m) $\frac{644}{5}$ n) $\frac{4}{15}$ o) 40

3. a) $\frac{1}{3}a^3\left(2\sqrt{2}-1\right)$ b) $\frac{1}{(n+1)(n+2)}$ c) $\ln|\sin x|+C$

4. a) $\sqrt{ax^2+2bx+c}+C$ b) $\frac{1}{ab}\arctan\left(\frac{bx}{a}\right)+C$

5. 7

6. $50\ln 37 \approx 181$ people

7. a) $\frac{7}{2\pi}$ liters b) 0 liters

8. a) $\frac{5324}{15}$ b) $\frac{-5}{36\pi}$