CEGEP CHAMPLAIN ST. LAWRENCE CHAMPLAIN REGIONAL COLLEGE

MATHEMATICS 201-NYB-05
Integral Calculus
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IV - The Fundamental Theorem of Calculus

1. Use Part I of the Fundamental Theorem of Calculus to find the derivative of the function.

a) $g(x)=\int_1^x \cos\sqrt{t}\,dt$

b) $g(x)=\int_4^x \frac{3du}{1+\sqrt[5]{u}}$

c) $g(x)=\int_{-4}^{x^3} e^{t^2}dt$

d) $g(x)=\int_x^3 \sin\left(t^2\right)dt$

e) $g(x)=\int_{\frac{1}{x}}^{x^4} \frac{1}{\cos t}dt$

f) $g(x)=\int_{\sec x}^{3x^2} \sqrt{1+t^2}\,dt$

2. Evaluate the definite integral.

a) $\int_1^3 x^5dx$

b) $\int_{-3}^0 \left(x^2-5x+1\right)dx$

c) $\int_1^4 \frac{1}{x^2}dx$

d) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin t\,dt$

e) $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \cos\theta\,d\theta$

f) $\int_4^9 x\sqrt{x}\,dx$

g) $\int_1^4 \left(3\sqrt{t}-\frac{4}{\sqrt{t}}+\sqrt[3]{t^2}\right)dt$

h) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(x+\frac{2}{\sin^2 x}\right)dt$

i) $\int_1^{\sqrt{3}} \frac{2}{1+x^2}dx$

j) $\int_1^4 \frac{x^3-1}{\sqrt{x}}dx$

k) $\int_0^2 (x+1)^3\,dx$

l) $\int_1^3 \left(e^x+\sqrt{\frac{5}{x}}\right)dx$

m) $\int_{-1}^6 f(x)dx$ where $f(x)=\begin{cases}3x^2-x & x\le 1\\ \frac{x+1}{x} & x>1\end{cases}$

n) $\int_{-1}^4 |2x-4|\,dx$

o) $\int_{-1}^2 \frac{4x^7+2x^4+4x^3+2}{x^4+1}dx$

p) $\int_2^4 e^{3x-6}dx$

q) $\int_{-1}^1 \frac{6x^3+17x^2+2x-14}{2x+3}dx$

3. Evaluate the following limit by first recognizing the sum as a Riemann sum.

a) $\lim_{n\to\infty}\left(\frac{1}{n^3}+\frac{4}{n^3}+\frac{9}{n^3}+\cdots+\frac{n^2}{n^3}\right)$

b) $\lim_{n\to\infty}\sum_{i=1}^n \frac{n}{n^2+i^2}$

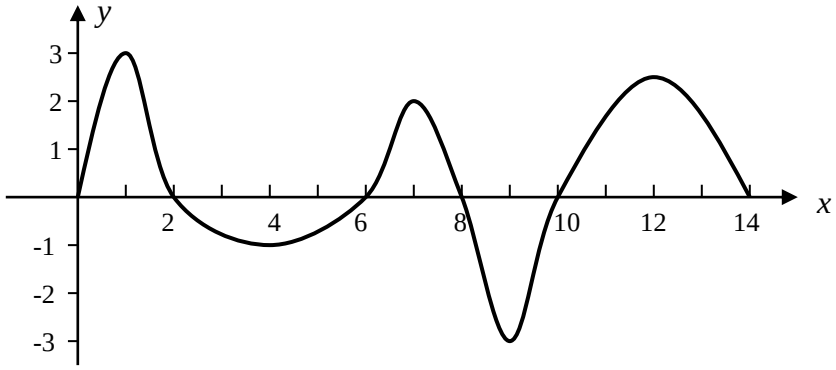
c) $\lim_{n\to\infty}\frac{1}{n^2}\left(\sqrt{n}+\sqrt{2n}+\sqrt{3n}+\sqrt{4n}+\ldots+\sqrt{n^2}\right)$

d) $\lim_{n\to\infty}\left(\frac{2n}{(n+2\cdot 1)^2}+\frac{2n}{(n+2\cdot 2)^2}+\frac{2n}{(n+2\cdot 3)^2}+\cdots+\frac{2n}{(n+2\cdot n)^2}\right)$

e) $\lim_{n\to\infty}\left(\frac{3e^{3+\frac{3}{n}}}{n}+\frac{3e^{3+\frac{6}{n}}}{n}+\frac{3e^{3+\frac{9}{n}}}{n}+\cdots+\frac{3e^{3+\frac{3n}{n}}}{n}\right)$

f) $\lim_{n\to\infty}\left(\frac{7}{\sqrt[3]{n^3+7n^2}}+\frac{7}{\sqrt[3]{n^3+14n^2}}+\frac{7}{\sqrt[3]{n^3+21n^2}}+\cdots+\frac{7}{\sqrt[3]{n^3+7n^3}}\right)$

4. Let $g(x)=\int_0^x f(t)\,dt$, where *f* is the function whose graph is given below.
   a) Find the values of *x* for which *g* has local maximum's and minimum's.
   b) Find the values of *x* for where *g* has a point of inflection.
   c) Sketch the graph of *g*.



5. Find the average value of the function on the given interval.
   a) $f(x)=4x^3-3x^2+2$ on $[-3,1]$ b) $f(x)=\cos x$ on $\left[\frac{-\pi}{4},\frac{\pi}{4}\right]$
   c) $f(x)=e^x$ on $[1,4]$

6. Find a value of *c* that satisfies the conclusion to the mean value theorem for integrals.
   a) $f(x)=4x^3-2$ on $[1,3]$ b) $f(x)=\frac{1}{1+x^2}$ on $[0,1]$

7. Suppose the (downward) velocity of a sky diver is given by $v(t)=10\left(1-e^{-t}\right)$ m/s for the first 5 seconds of a jump. Compute the distance fallen.

8. The velocity (in meters per second) of a particle moving along a line is given by $v(t)=-t^2+4t$.
   a) Find the displacement by the particle during the time interval $1\le t\le 6$.
   b) Find the distance traveled by the particle during the time interval $1\le t\le 6$.

9. The acceleration function (in m/s^2) for a particle moving along a line is given by $a(t) = 2t + 3$. If the initial velocity is –3 m/s, find the distance traveled during the first 2 seconds.

10. Suppose that water can flow in and out of a storage tank. The net rate of change (that is the rate in minus the rate out) of water is $f'(t) = 20\left(t^2 - 1\right)$ liters per minute.
    a) For $0 \le t \le 3$, find the change in the amount of water in the reservoir.
    b) If the tank has 200 liters of water at time $t = 0$, determine how many liters are in the tank at $t = 3$.

11. The rate of change of the weight of a laboratory mouse can be modeled by the equation $W(t) = \frac{37}{5t}$ grams per week, where *t* is the age of the mouse, in weeks, beyond 2 weeks.
    a) What is the weight increase of the mouse between week 2 and week 8?
    b) If the mouse weighs 26 grams after 8 weeks, what was its weight after 2 weeks?

12. After long study, tree scientists conclude that a eucalyptus tree will grow at the rate of $H(t) = 0.6 + 12t^{-4}$ meters per year, where *t* is in years.
    a) Find the number of meters that the tree will grow in the second year.
    b) Find the number of meters the tree will grow in the third year.

13. Pollution from a factory is entering a lake. The rate of concentration of the pollutant at time *t* is given by
$$P'(t) = 140t^{\frac{5}{2}}$$
where *t* is the number of years since the factory started introducing pollutants into the lake. Ecologists estimate that the lake can accept a total level of pollution of 4850 units before all the fish life in the lake ends. Can the factory operate for 4 years without killing all the fish in the lake?

14. For a certain drug, the rate of reaction in appropriate units is given by
$$R'(t) = \frac{5}{t} + \frac{2}{t^2}$$
where *t* is time measured in hours after the drug is administered. Find the total reaction to the drug over the following time periods.
    a) from $t = 1$ to $t = 12$
    b) from $t = 12$ to $t = 24$

15. If $h'(t)$ represent the rate of growth of a fir tree in meters per year, what does $\int_4^{10} h'(t)\,dt$ represent? What are the units?

Answers

1. a) $\cos\sqrt{x}$ b) $\dfrac{3}{1+\sqrt[5]{x}}$ c) $3x^2e^{x^6}$

   d) $-\sin\left(x^2\right)$ e) $\dfrac{4x^3}{\cos\left(x^4\right)}+\dfrac{1}{x^2\cos\left(\frac{1}{x}\right)}$ g) $6x\sqrt{1+9x^4}-\sec x\tan x\sqrt{1+\sec^2 x}$

2. a) $\frac{364}{3}$ b) $\frac{69}{2}$ c) $\frac{3}{4}$ d) 0 e) $\sqrt{2}$

   f) $\frac{422}{5}$ g) $\frac{27}{5}+\frac{24}{5}\sqrt[3]{2}$ h) $\frac{\pi^2}{9}+2\sqrt{3}$ i) $\frac{\pi}{6}$ j) $\frac{240}{7}$

   k) 20 l) $e^3+2\sqrt{15}-e-2\sqrt{5}$ m) $7+\ln 6$ n) 13

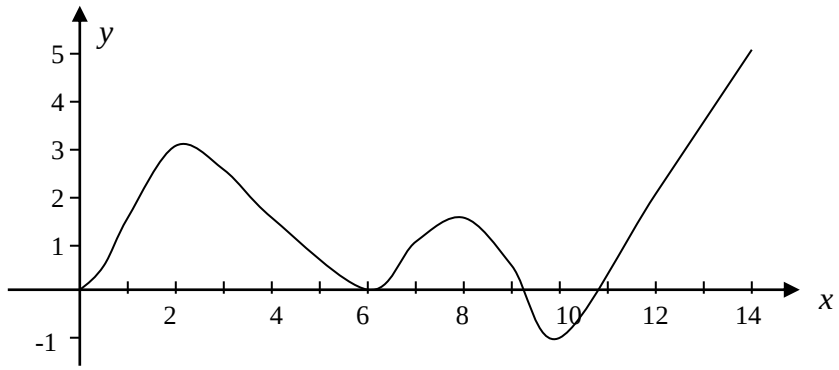
   o) 21 p) $\frac{1}{3}e^6-\frac{1}{3}$ q) $\frac{1}{2}\ln 5-8$

3. a) $\frac{1}{3}$ b) $\frac{\pi}{4}$ c) $\frac{2}{3}$ d) $\frac{2}{3}$ e) e^6-e^3 f) $\frac{9}{2}$

4. a) maximum at $x=2$, 8 and 14 minimum at $x=6,\ 10$

   b) Inflection points at $x=1$, 4, 7, 9, 12

   c)



5. a) -25 b) $\frac{2\sqrt{2}}{\pi}$ c) $\frac{1}{3}e^4-\frac{1}{3}e$

6. a) $\sqrt[3]{10}$ b) $\frac{\sqrt{4\pi-\pi^2}}{\pi}$

7. $40+\frac{10}{e^5}\approx 40.07$ m

8. a) $\frac{-5}{3}$ m b) $\frac{59}{3}$ m

9. $\frac{8}{3}$ m

10. a) 120 liters b) 320 liters

11. a) $\frac{74}{5}\ln 2$ g b) $\left(26-\frac{72}{5}\ln 2\right)$ g

12. a) $\frac{41}{10}$ m b) $\frac{257}{270}\approx 0.952$ m

13. No. The pollution in the lake will increase by 5120 units in the first four years.

14. a) $10\ln 2+5\ln 3+\frac{11}{6}\approx 14.26$ b) $5\ln 2+\frac{1}{12}\approx 3.55$

15. The increase in the height of the tree (in meters) between the ages of 4 and 10.