MATHEMATICS 201-NYB-05
Integral Calculus
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II - Sigma Notation and Areas

1. Write the sum in expanded form.

a) $\sum_{i=1}^{5}(2i+1)$

b) $\sum_{k=0}^{4}\frac{(-1)^k}{k^2+1}$

c) $\sum_{i=2}^{4}5^i$

d) $\sum_{i=1}^{n}i^4$

2. Write the sum in sigma notation.

a) $1+2+3+4+...+10$

b) $\frac{1}{3}+\frac{2}{4}+\frac{3}{5}+...+\frac{18}{20}$

c) $1+3+9+27+81+243$

d) $1-3+5-7+9-11$

3. Find the value of the sum.

a) $\sum_{i=3}^{6}(4i-3)$

b) $\sum_{i=1}^{25}(-1)^i$

c) $\sum_{i=1}^{15}\left(2i^2-3i+4\right)$

d) $\sum_{i=1}^{10}\left(i^3-5i+3\right)$

e) $\sum_{i=1}^{n}(i^2+2i-3)$

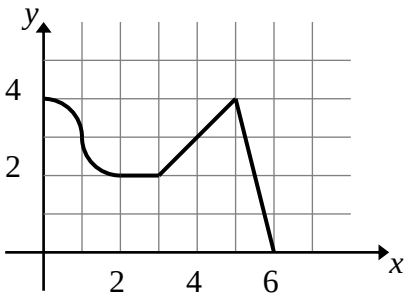
f) $\sum_{i=1}^{n}(i-1)(i+2)$

g) $\sum_{i=1}^{n}(2i^3-i+1)$

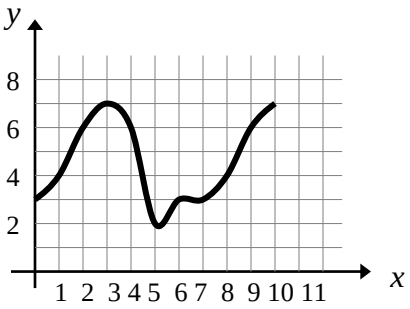
h) $\sum_{i=1}^{n}\left(\frac{1}{n}-\frac{3i}{n}\right)$

4. Consider the function *f* whose graph is given. Estimate the area under *f* on the given interval using the given number of rectangles and (i) right endpoints (ii) left endpoints.

a) on $[0,6]$ using 6 rectangles.



b) on $[2,10]$ using 4 rectangles.



5. Consider the area under the graph of $f(x)=25-x^2$ from $x=0$ to $x=5$.
   a) Approximate this area using five rectangles and the right endpoints of the subintervals. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?
   b) Same as (a) but using the left endpoints of the subintervals.

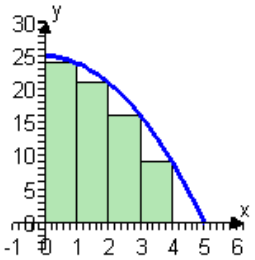
6. Consider the area under the graph of $f(x)=2^x$ from $x=-1$ to $x=7$.
   a) Approximate this area using four rectangles and the right endpoints of the subintervals. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?
   b) Same as (a) but using the left endpoints of the subintervals.

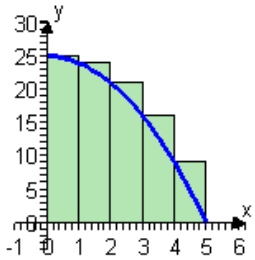
7. Find the area under the given curve from $x=a$ to $x=b$. Use equal subintervals and take x_i^* to be the right-hand endpoint of the ith subinterval.
   a) $y=2x+1$ $a=0,\ b=4$
   b) $y=3x-4$ $a=2,\ b=5$
   c) $y=x^2+3x+1$ $a=1,\ b=3$
   d) $y=16-x^2$ $a=-4,\ b=4$
   e) $y=(x-5)(x-3)$ $a=-2,\ b=3$
   f) $y=x^3-x^2+4$ $a=-1,\ b=3$

8. Find the area under the curve $y=x^2-1$ from $a=1$ to $b=5$ using equal subintervals and taking x_i^* to be
   a) the right-hand endpoints of the subintervals.
   b) the left-hand endpoints of the subintervals.
   c) the midpoints of the subintervals.

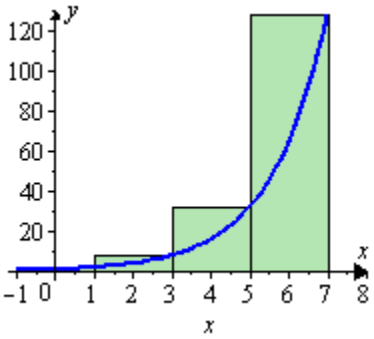
Answers

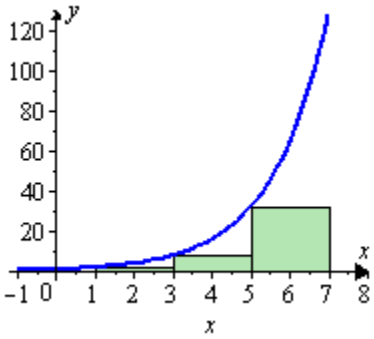
1. a) $3+5+7+9+11$ b) $1-\frac{1}{2}+\frac{1}{5}-\frac{1}{10}+\frac{1}{17}$
   c) $5^2+5^3+5^4$ d) $1+2^4+3^4+4^4+\ldots+n^4$
2. a) $\sum_{i=1}^{10} i$ b) $\sum_{i=1}^{18} \frac{i}{i+2}$ c) $\sum_{i=0}^{5} 3^i$ d) $\sum_{i=1}^{6} (2i-1)(-1)^{i-1}$
3. a) 60 b) –1 c) 2180 d) 2780 e) $\frac{n(2n^2+9n-11)}{6}$
   f) $\frac{n(n^2+3n-4)}{3}$ g) $\frac{n(n^3+2n^2+1)}{2}$ h) $\frac{-3n-1}{2}$
4. a) 14, 18 b) 40, 38
5. a) 70 underestimate b) 95 overestimate





6. a) 340 overestimate b) 85 underestimate





7. a) $\lim_{n\to\infty} \frac{4(5n+4)}{n} = 20$ b) $\lim_{n\to\infty} \frac{3(13n+9)}{2n} = \frac{39}{2}$
   c) $\lim_{n\to\infty} \frac{2(2n+1)(17n+2)}{3n^2} = \frac{68}{3}$ d) $\lim_{n\to\infty} \frac{256(n-1)(n+1)}{3n^2} = \frac{256}{3}$
   e) $\lim_{n\to\infty} \frac{25(16n-5)(n-1)}{3n^2} = \frac{200}{3}$ f) $\lim_{n\to\infty} \frac{8(10n^2+15n+8)}{3n^2} = \frac{80}{3}$
8. a) $\lim_{n\to\infty} \frac{16(n+1)(7n+2)}{3n^2} = \frac{112}{3}$ b) $\lim_{n\to\infty} \frac{16(n-1)(7n-2)}{3n^2} = \frac{112}{3}$
   c) $\lim_{n\to\infty} \frac{16(7n^2-1)}{3n^2} = \frac{112}{3}$