MATHEMATICS 201-NYB-05
Integral Calculus
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I - Antiderivatives

1. Find the antiderivatives.

a) $\int \left(6x^2 - 8x + 3\right) dx$

b) $\int \left(x - \sqrt{x}\right) dx$

c) $\int \left(\frac{1}{x^2} - \frac{1}{x^3}\right) dx$

d) $\int \left(x\sqrt{x} + \sqrt[5]{x^3}\right) dx$

e) $\int \frac{x^3 + 2x^2}{\sqrt{x}}\, dx$

f) $\int \left(\frac{2}{x} + 3e^x\right) dx$

g) $\int \left(1 + x^2\right)(2x + 1)\, dx$

h) $\int \frac{5}{1 + x^2}\, dx$

i) $\int \frac{\left(\sqrt{x} + \sqrt[3]{x}\right)\left(\sqrt[4]{x} + \sqrt[5]{x}\right)}{\sqrt[6]{x}} dx$

j) $\int \left(4\sin x - 3\cos x\right) dx$

k) $\int \frac{x^4 + x^2 + 3}{x^2 + 1} dx$

l) $\int \frac{\sin x}{\cos^2 x}\, dx$

m) $\int \left(\frac{\sec x \tan x + \csc x}{\sin x}\right) dx$

n) $\int \frac{e^{-x} + 1}{e^{-x}}\, dx$

o) $\int \frac{x - 1}{\sqrt{x} + 1} dx$

p) $\int \frac{x^5 - x^4 + 2x^3 + x^2 + 6}{x^2 + 2} dx$

2. Find $f(x)$.

a) $f''(x) = 24x^2 - 4x + 2$

b) $f''(x) = 5x^4 + \frac{1}{\sqrt{x}}$

c) $f'''(x) = e^x$

d) $f'(x) = 1 + \frac{1}{x^2}$ $f(1) = 2$

e) $f''(x) = 12x^3 + 6x$ $f(1) = 2$ $f'(1) = 5$

f) $f''(x) = x^2 + \sin x$ $f(0) = -1$ $f'(0) = 3$

g) $f''(x) = 18x^2 - 6$ $f(1) = 2$ $f(3) = 5$

3. Find y.

a) $\frac{dy}{dx} = x^2 + 1$

b) $\frac{dy}{dx} = y^2 + 1$

c) $\frac{dy}{dx} = xy$ $y(0) = 3$

4. A particle is moving with the given data. Find the position of the particle.
   a) $v(t) = 5 - 4t \qquad s(0) = -4$
   b) $a(t) = \sin t + \cos t \qquad s(0) = 0 \qquad v(0) = 5$
   c) $a(t) = 3t^2 + 5t + 1 \qquad s(0) = 2 \qquad s(1) = 4$

5. A stone is dropped from the upper observation deck of the CN Tower, 450 m above ground.
   a) Find the distance of the stone above the ground at time *t*.
   b) How long does the stone take to hit the ground?
   c) What is the velocity of the stone when it hits the ground.

6. Same as 4 but the stone is thrown downward with a velocity of 4 m/s.

7. Show that for motion in a straight line with constant acceleration *a*, initial velocity v_0 and initial displacement s_0, the displacement after time *t* is
$$s = \tfrac{1}{2}at^2 + v_0 t + s_0$$

8. The make of a certain automobile advertises that it takes 13 seconds to accelerate from 25m/s to 80 m/s. Assuming constant acceleration, find
   a) The acceleration in m/s^2.
   b) The distance the car travels during the 13 seconds.

9. A car braked with a constant deceleration of 15 m/s^2, producing skid marks measuring 60 m before coming to a stop. How fast was the car traveling when the brakes were first applied?

10. The rate of growth $\dfrac{dP}{dt}$ of a population of bacteria is proportional to the square root of *t*, where *P* is the population size and *t* is the time in days $(0 \le t \le 10)$. That is
$$\frac{dP}{dt} = k\sqrt{t}$$
The initial size of the population is 500. After 1 day, the population has grown to 600. Estimate the population after 7 days.

11. Ammonium cyanate, NH_4NCO, rearranges in water to give urea, $(NH_2)_2CO$.
$$NH_4NCO \rightarrow (NH_2)_2CO$$
The rate equation is given by
$$-\frac{d\left[NH_4NCO\right]}{dt} = k\left[NH_4NCO\right]^2$$
If it took 118 minutes for the concentration of NH_4NCO to drop from 0.500 mol/L to 0.300 mol/L, what will be the concentration after 3 hours (for an original concentration of 0.500 mol/L)?

Answers

1. a) $2x^3-4x^2+3x+C$ b) $\frac{x^2}{2}-\frac{2}{3}x^{\frac{3}{2}}+C$ c) $-\frac{1}{x}+\frac{1}{2x^2}+C$

   d) $\frac{2}{5}x^{\frac{5}{2}}+\frac{5}{8}x^{\frac{8}{5}}+C$ e) $\frac{2}{7}x^{\frac{7}{2}}+\frac{4}{5}x^{\frac{5}{2}}+C$ f) $2\ln|x|+3e^x+C$

   g) $\frac{x^4}{2}+\frac{x^3}{3}+x^2+x+C$ h) $5\arctan x+C$

   i) $f(x)=\frac{12}{17}x^{\frac{17}{12}}+\frac{15}{23}x^{\frac{23}{15}}+\frac{12}{19}x^{\frac{19}{12}}+\frac{30}{41}x^{\frac{41}{30}}$ j) $-4\cos x-3\sin x+C$

   k) $\frac{1}{3}x^3+3\arctan x+C$ l) $\sec x+C$ m) $\tan x-\cot x+C$

   n) $x+e^x+C$ o) $\frac{2}{3}x^{\frac{3}{2}}-x+C$ p) $\frac{1}{4}x^4-\frac{1}{3}x^3+3x+C$

2. a) $f(x)=2x^4-\frac{2}{3}x^3+x^2+Cx+D$ b) $f(x)=\frac{1}{6}x^6+\frac{4}{3}x^{\frac{3}{2}}+Cx+D$

   c) $f(x)=e^x+Cx^2+Dx+E$ d) $f(x)=x-\frac{1}{x}+2$ e) $f(x)=\frac{3}{5}x^5+x^3-x+\frac{7}{5}$

   f) $f(x)=\frac{1}{12}x^4-\sin x+4x-1$ g) $f(x)=\frac{3}{2}x^4-3x^2-\frac{93}{2}x+50$

3. a) $y=\frac{1}{3}x^3+x+C$ b) $y=\tan(x+C)$ c) $y=3e^{\frac{1}{2}x^2}$

4. a) $s(t)=-2t^2+5t-4$ b) $s(t)=-\sin t-\cos t+6t+1$

   c) $s(t)=\frac{1}{4}t^4+\frac{5}{6}t^3+\frac{1}{2}t^2+\frac{5}{12}t+2$

5. a) $s(t)=450-4.9t^2$ b) $\frac{30\sqrt{5}}{7}$ s c) $-42\sqrt{5}$ m/s

6. a) $s(t)=450-4t-4.9t^2$ c) $\frac{450}{49}$ s d) –94 m/s

8. a) $\frac{55}{13}$ m/s^2 b) $\frac{1365}{2}$ m

9. $30\sqrt{2}$ m/s

10. $700\sqrt{7}+500\approx 2352$

11. $\frac{59}{238}$ mol/L