



MATHEMATICS 201-NYA-05

Differential Calculus

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X - Higher Order Derivatives

1. Find the second derivative for each of the following functions.

a) $f(x) = x^2(x^2 + 5)$

b) $f(t) = \sqrt{t^2 + 1}$

c) $f(x) = \sin\left(\frac{x}{x+1}\right)$

d) $f(x) = \left(x + \frac{1}{x}\right)^2$

e) $f(\theta) = \sqrt{1 + \cos 3\theta}$

f) $f(x) = (x + \cot 2x)^4$

g) $f(t) = \frac{2t-5}{4t}$

h) $f(x) = (x^3 - 5)(4x + 3)$

i) $f(z) = \frac{1-z}{1+z}$

j) $f(x) = x \tan\left(\frac{1}{x}\right)$

2. Find $\frac{d^3 y}{dx^3}$

a) $y = \frac{1}{x^4} - 3x^6 + \sqrt{2x}$

b) $y = \sin x \cos x$

3. Find $\frac{d^{81} y}{dx^{81}}$ if $y = \sin x$. (Hint: look for a pattern in the first couple of derivatives. Do NOT derive eighty one times!.)

4. Find $\frac{d^{100} y}{dx^{100}}$ if $y = \cos x$. (Hint: look for a pattern in the first couple of derivatives. Do NOT derive one hundred times!.)

5. Find $f^{(n)}(x)$.

a) $f(x) = x^n$

b) $f(x) = \frac{1}{2x+1}$

c) $f(x) = \frac{1}{\sqrt{x}}$

d) $f(x) = \sin x \cos x$

6. Find $\frac{d^2 y}{dx^2}$ in terms of x and/or y .

a) $x^4 + y^4 = 64$

b) $y + \sin y = x$

c) $2xy - y^2 = 3$

d) $x = y \cos y$

7. Find $\frac{d^3y}{dx^3}$ in terms of x and/or y for $x^2 + y^2 = 4$.
8. A particle is moving along a linear scale according to the equation $s = 3t^4 - 4t^3 + 10t, t \geq 0$, where s is the distance in meters from the origin at the end of t seconds.
- What is the instantaneous acceleration when $t = 0$? $t = 3$?
 - Find the time when the instantaneous acceleration is zero?
9. A particle is moving along a linear scale according to the equation $s = 5t + \frac{2}{t+1}, t \geq 0$, where s is the distance in meters from the origin at the end of t seconds.
- What is the instantaneous acceleration when $t = 0$? $t = 9$?
 - Find the time when the instantaneous acceleration is zero.
10. Prove that $(f \cdot g)'' = f'' \cdot g + 2f' \cdot g' + f \cdot g''$

