CEGEP CHAMPLAIN ST. LAWRENCE CHAMPLAIN REGIONAL COLLEGE

MATHEMATICS 201-NYA-05
Differential Calculus
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VIII – The Chain Rule

1. Differentiate the function.

a) $f(x)=\left(3x^2-4x\right)^{15}$

b) $f(x)=\left(\sqrt{x}+\frac{1}{x}\right)^{12}$

c) $f(x)=\dfrac{1}{\left(2x-5\right)^7}$

d) $f(x)=\sqrt[5]{x^3-2x+1}$

e) $f(t)=\sqrt{t^4-t+\sqrt{3t}}$

f) $f(x)=2\cos 4x$

g) $f(x)=\tan\sqrt{x}$

h) $f(x)=3\cos^5 x$

i) $f(x)=3\cos^5(10x)$

j) $f(x)=2\sec^2\left(x^6\right)$

k) $f(x)=\sin(\cos x)$

l) $f(x)=\dfrac{\sin 3x}{1+\cos 5x}$

m) $f(z)=\csc^2(12z)-\cot^2(12z)$

n) $f(x)=\cos^3\left(\dfrac{x}{x+1}\right)$

o) $f(x)=\left(x^4-\sec\left(4x^2-2\right)\right)^{-4}$

p) $f(x)=\sin^2 x\sqrt{x+\cos^3(5x)}$

q) $f(x)=\left(3x^2-1\right)^4\left(2x+4\right)^3$

r) $f(x)=\left(2x-5\right)^2\left(x^2+4\right)^3$

s) $f(x)=\dfrac{\left(2x+3\right)^3}{\left(4x^2-1\right)^8}$

t) $f(x)=\left(\dfrac{x^2+x}{1-2x}\right)^4$

u) $f(x)=\sin^3\left(x^2\right)\cos^4\left(x^2\right)$

v) $f(x)=\sqrt{\csc^3(5x)}\cot^4(5x)$

w) $f(x)=\left(2x-1\right)^{\frac{3}{2}}\left(5x+2\right)^{\frac{1}{3}}$

x) $f(t)=\sqrt{t}\sec\sqrt{t}-\frac{1}{t}\tan\frac{1}{t}$

y) $f(x)=\dfrac{\sqrt{3x-1}}{\sqrt[3]{2x+5}}$

z) $f(z)=\dfrac{1}{\sqrt{z^2+1}}-\sqrt{1+\sin^2 z}+\sqrt[5]{\pi}$

2. Find $\dfrac{dy}{dx}$.

a) $y=x^2\sin^3(2x)$

b) $y=\tan^4\left(\sqrt{3x-1}\right)$

c) $y=\sin(\cos(\tan 5x))$

d) $y=\sec^2 x\tan^3 x$

e) $y=\dfrac{1+\csc\left(x^2\right)}{1-\cot\left(x^2\right)}$

f) $y=\left(2x+\frac{1}{x}\right)^2\left(3x-1\right)^5$

g) $y=\dfrac{\left(4x^2-1\right)^5}{\left(1-2x\right)^3}$

h) $y=\left(\dfrac{16x+1}{2x-1}\right)^{-4}$

i) $y=\dfrac{x^2+1}{x\left(x+2\right)^3}$

j) $y=\sin^4\left(3x^2\right)\cos^3\left(3x^2\right)$

k) $y=\left(4x+3\right)^4\sqrt[3]{2x+1}$

3. Find an equation for the tangent line to the graph at the specified point.
   a) $y=\sec^3\left(\frac{\pi}{2}-x\right)$ $\quad x=\frac{-\pi}{2}$
   b) $y=\left(x-\frac{1}{x}\right)^4$ $\quad x=2$

4. Find all points on the graph of the function at which the tangent line is horizontal.
   a) $y=\left(3x-5\right)^4\left(2x+1\right)^5$
   b) $y=\dfrac{\left(x^2-3\right)^4}{\left(x^2+3\right)^5}$

5. Find the equation of all tangent lines to the graph of $f\left(x\right)=\left(3x-1\right)^4$ that are parallel to $y=12x+5$.

6. Find the equation of all tangent lines to the graph of $f\left(x\right)=\dfrac{-9}{\left(3x+2\right)^3}$ that are parallel to $y=16x-2$.

7. Find $\dfrac{dy}{dx}$ if $y=\left(\dfrac{ax+b}{cx+d}\right)^5$, where *a*, *b*, *c* and *d* are constants.

8. The electric field intensity *E* on the axis of a uniformly charged ring at a point *x* units from the center of the ring is given by the formula

$$E=\frac{Qx}{\left(a^2+x^2\right)^{\frac{3}{2}}}$$

where *a* and *Q* are constants. Find a formula for the rate of change of field intensity *E* with respect to distance *x* along the axis.

9. The strength of a person's reaction to a certain dug is given by $R\left(Q\right)=Q\sqrt{C-\frac{1}{3}Q}$ where Q represents the quantity of the drug given to the patient and C is a constant.
   a) The derivative $R'\left(Q\right)$ is called the *sensitivity* to the drug. Find $R'\left(Q\right)$.
   b) Find the sensitivity to the drug if $C=59$ and a patient is given 87 units of the drug.

ANSWERS

1. a) $f'(x)=30(3x-2)(3x^2-4x)^{14}$

b) $f'(x)=12\left(\frac{1}{2\sqrt{x}}-\frac{1}{x^2}\right)\left(\sqrt{x}+\frac{1}{x}\right)^{11}$

c) $f'(x)=\frac{-14}{(2x-5)^8}$

d) $f'(x)=\frac{3x^2-2}{5\left(x^3-2x+1\right)^{\frac{4}{5}}}$

e) $f'(t)=\frac{8t^{\frac{7}{2}}-2\sqrt{t}+\sqrt{3}}{4\sqrt{t^5-t^2+\sqrt{3}t^{\frac{3}{2}}}}$

f) $f'(x)=-8\sin 4x$

g) $f'(x)=\frac{\sec^2\sqrt{x}}{2\sqrt{x}}$

h) $f'(x)=-15\cos^4 x\sin x$

i) $f'(x)=-150\cos^4(10x)\sin(10x)$

j) $f'(x)=24x^5\sec^2\left(x^6\right)\tan\left(x^6\right)$

k) $f'(x)=-\sin x\cos(\cos x)$

l) $f'(x)=\frac{3\cos 3x+3\cos 3x\cos 5x+5\sin 3x\sin 5x}{(1+\cos 5x)^2}$

m) $f'(z)=0$

n) $f'(x)=\frac{-3\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right)}{(x+1)^2}$

o) $f'(x)=\frac{-16x^3+32x\sec\left(4x^2-2\right)\tan\left(4x^2-2\right)}{\left(x^4-\sec\left(4x^2-2\right)\right)^5}$

p) $f'(x)=2\sin x\cos x\sqrt{x+\cos^3(5x)}+\frac{\sin^2 x\left(1-15\cos^2(5x)\sin(5x)\right)}{2\sqrt{x+\cos^3(5x)}}$

q) $f'(x)=6\left(11x^2+16x-1\right)(2x+4)^2\left(3x^2-1\right)^3$

r) $f'(x)=2\left(8x^2-15x+8\right)(2x-5)\left(x^2+4\right)^2$

s) $f'(x)=\frac{-2(2x+3)^2\left(52x^2+96x+3\right)}{\left(4x^2-1\right)^9}$

t) $f'(x)=\frac{-4\left(x^2+x\right)^3\left(2x^2-2x-1\right)}{(1-2x)^5}$

u) $f'(x)=6x\sin^2\left(x^2\right)\cos^5\left(x^2\right)-8x\sin^4\left(x^2\right)\cos^3\left(x^2\right)$

v) $f'(x)=\frac{-15}{2}\csc^{\frac{3}{2}}(5x)\cot^5(5x)-20\csc^{\frac{7}{2}}(5x)\cot^3(5x)$

w) $f'(x)=\frac{(55x+13)\sqrt{2x-1}}{3(5x+2)^{\frac{2}{3}}}$

x) $f'(t)=\dfrac{\sec\sqrt{t}}{2\sqrt{t}}+\frac{1}{2}\sec\sqrt{t}\tan\sqrt{t}+\dfrac{\sec^2\frac{1}{t}+t\tan\frac{1}{t}}{t^3}$

y) $f'(x)=\dfrac{6x+49}{6(3x-1)^{\frac{1}{2}}(2x+5)^{\frac{4}{3}}}$

z) $f'(z)=\dfrac{-z}{(z^2+1)^{\frac{3}{2}}}-\dfrac{\sin z\cos z}{\sqrt{1+\sin^2 z}}$

2. a) $\dfrac{dy}{dx}=2x\sin^3(2x)+6x^2\sin^2(2x)\cos(2x)$

b) $\dfrac{dy}{dx}=\dfrac{6\tan^3\sqrt{3x-1}\sec^2\sqrt{3x-1}}{\sqrt{3x-1}}$

c) $\dfrac{dy}{dx}=-5\cos(\cos(\tan 5x))\sin(\tan(5x))\sec^2(5x)$

d) $\dfrac{dy}{dx}=2\sec^2 x\tan^4 x+3\sec^4 x\tan^2 x$

e) $\dfrac{dy}{dx}=\dfrac{-2x\left(\csc(x^2)\cot^2(x^2)-\csc(x^2)\cot(x^2)-\csc^2(x^2)-\csc^3(x^2)\right)}{\left(1-\cot(x^2)\right)^2}$

f) $\dfrac{dy}{dx}=\dfrac{(2x^2+1)(3x-1)^4(42x^3-4x^2+9x+2)}{x^3}$

g) $\dfrac{dy}{dx}=-2(2x+1)^4(2x-1)(14x-3)$

h) $\dfrac{dy}{dx}=\dfrac{72(2x-1)^3}{(16x+1)^5}$

i) $\dfrac{dy}{dx}=\dfrac{2x^3-2x^2+4x+2}{x^2(x+2)^4}$

j) $\dfrac{dy}{dx}=24x\sin^3(3x^2)\cos^4(3x^2)-18x\sin^5(3x^2)\cos^2(3x^2)$

k) $\dfrac{dy}{dx}=\dfrac{2(52x+27)(4x+3)^3}{3(2x+1)^{\frac{2}{3}}}$

3. a) $y=-1$ b) $y=\frac{135}{8}x-\frac{459}{16}$

4. a) $x=\frac{19}{27},\frac{5}{3},\frac{-1}{2}$ b) $x=0,\pm\sqrt{3},\pm 3\sqrt{3}$

5. $y=12x-7$

6. $y=16x$ and $y=16x+\frac{64}{3}$

7. $\dfrac{dy}{dx}=\dfrac{5(ax+b)^4(ad-bc)}{(cx+d)^6}$

8. $\dfrac{dE}{dx}=\dfrac{Q(a^2-2x^2)}{(a^2+x^2)^{\frac{5}{2}}}$

9. a) $R'(Q)=\dfrac{6C-3Q}{2\sqrt{9C-3Q}}$ b) $\dfrac{31\sqrt{30}}{60}$