



## MATHEMATICS 201-NYA-05

Differential Calculus

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# VII – Derivatives of Trigonometric Functions

1. Convert from degrees to radians.

a)  $135^\circ$                       b)  $900^\circ$                       c)  $-315^\circ$

2. Convert from radians to degrees.

a)  $\frac{8\pi}{3}$                       b)  $\frac{-7\pi}{12}$                       c) 3

3. Evaluate exactly (without the use of a calculator).

a)  $\sin\left(\frac{2\pi}{3}\right)$               b)  $\tan\left(\frac{-\pi}{4}\right)$               c)  $\cos\left(\frac{5\pi}{6}\right)$               d)  $\sin\left(\frac{5\pi}{2}\right)$   
e)  $\sec\left(\frac{7\pi}{4}\right)$               f)  $\cot \pi$                       g)  $\csc\left(\frac{7\pi}{6}\right)$               h)  $\cot\left(\frac{3\pi}{4}\right)$

4. Use identities to evaluate exactly.

a)  $\sin(75^\circ)$                       b)  $\cos\left(\frac{\pi}{12}\right)$                       c)  $\cos(165^\circ)$

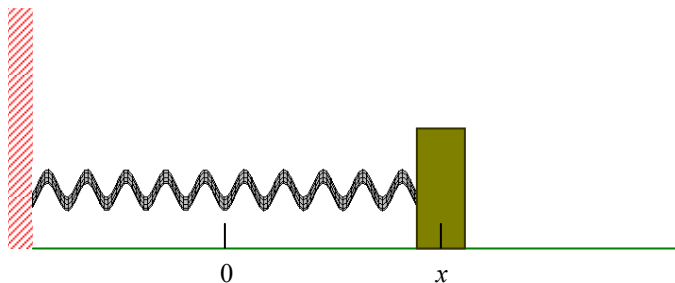
5. Find the limit.

a)  $\lim_{x \rightarrow 0} \frac{\sin 4x}{2x}$                       b)  $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 7x}$                       c)  $\lim_{\theta \rightarrow 0} \frac{\sin(\cos \theta)}{\sec \theta}$   
d)  $\lim_{x \rightarrow 0} \frac{\sin x - \cos x \sin x}{x^2}$                       e)  $\lim_{t \rightarrow 0} \frac{\tan 2t}{3t}$                       f)  $\lim_{x \rightarrow 0} \frac{\cot x}{\csc 2x}$   
g)  $\lim_{x \rightarrow 0} (\sin 3x \csc 12x)$                       h)  $\lim_{x \rightarrow 0} \frac{2x + \sin x}{x}$                       i)  $\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2 + 2x - 3}$

6. Differentiate the function.

a)  $f(x) = 2 \cos x - 5 \sin x$                       b)  $f(x) = \frac{\sin x}{x}$                       c)  $f(x) = \sec x - 5 \tan x$   
d)  $f(x) = \sec x \tan x$                       e)  $f(t) = t^3 \csc t - t \cot t$                       f)  $f(x) = \frac{\cot x}{1 + \csc x}$   
g)  $f(x) = \frac{\csc x}{\tan x}$                       h)  $f(x) = \frac{x^2 + 1}{\cos x - 1}$                       i)  $f(x) = \frac{1 - \sin x}{1 + 2 \sin x}$   
j)  $f(x) = \sqrt{x} \sin x + \frac{\cos x}{\sqrt{x}}$                       k)  $f(\theta) = \frac{3 \cos \theta}{2 \cos \theta - \sin \theta}$                       l)  $f(x) = \frac{\sin x \sec x}{1 + x \tan x}$

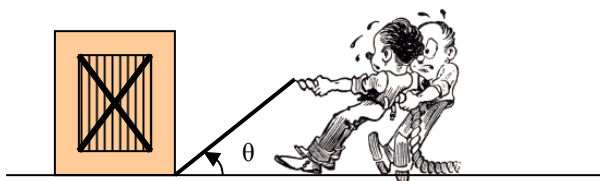
7. Find the equation for the tangent and normal lines to the graph of each function at the given point.
- $f(x) = 2 \sin x$  at  $(\frac{\pi}{6}, 1)$ .
  - $f(x) = 3 \tan x$  at  $(\frac{3\pi}{4}, -3)$ .
  - $f(x) = x + \cos x$  at  $(\pi, \pi - 1)$ .
  - $f(x) = \sec x + \csc x$  at  $(\frac{\pi}{4}, 2\sqrt{2})$ .
  - $f(x) = -2 \cot x$  at  $(\frac{5\pi}{6}, 2\sqrt{3})$ .
8. For what values of  $x$  does the graph of  $f(x) = x + 2 \cos x$  have a horizontal tangent?
9. A mass on a spring vibrates horizontally on a smooth level surface (see the figure). Its equation of motion is  $x(t) = 8 \sin t$ , where  $t$  is in seconds and  $x$  in centimeters.
- Find the velocity at time  $t$ .
  - Find the position and velocity of the mass at time  $t = \frac{2\pi}{3}$ . In what direction is it moving at that time?



10. The force  $F$  (in Newtons) acting at an angle  $\theta$  with the horizontal that is needed to drag a crate with weight  $W$  along a horizontal surface at a constant velocity is given by

$$F = \frac{\mu W}{\cos \theta + \mu \sin \theta}$$

where  $\mu$  is a constant called the coefficient of sliding friction between the crate and the surface (see the figure). Suppose that the crate weighs 50 N and that  $\mu = 0.4$ . Find  $\frac{dF}{d\theta}$  when  $\theta = 30^\circ$ .



## ANSWERS

1. a)  $\frac{3\pi}{4}$     b)  $5\pi$     c)  $\frac{-7\pi}{4}$
2. a)  $480^\circ$     b)  $-105^\circ$     c)  $\frac{540}{\pi}^\circ$
3. a)  $\frac{\sqrt{3}}{2}$     b)  $-1$     c)  $\frac{-\sqrt{3}}{2}$     d)  $1$     e)  $\sqrt{2}$     f)  $\cancel{2}$     g)  $-2$     h)  $-1$
4. a)  $\frac{\sqrt{6+\sqrt{2}}}{4}$     b)  $\frac{\sqrt{6-\sqrt{2}}}{4}$     c)  $\frac{-\sqrt{6-\sqrt{2}}}{4}$
5. a)  $2$     b)  $\frac{3}{7}$     c)  $\sin 1$     d)  $0$     e)  $\frac{2}{3}$     f)  $2$     g)  $\frac{1}{4}$     h)  $3$     i)  $\frac{1}{4}$
6. a)  $f'(x) = -2\sin x - 5\cos x$     b)  $f'(x) = \frac{x \cos x - \sin x}{x^2}$   
 c)  $f'(x) = \sec x \tan x - 5\sec^2 x$     d)  $f'(x) = \sec x \tan^2 x + \sec^3 x$   
 e)  $f'(t) = 3t^2 \csc t - t^3 \csc t \cot t - \cot t + t \csc^2 t$   
 f)  $f'(x) = \frac{-\csc^3 x - \csc^2 x + \csc x \cot^2 x}{(1 + \csc x)^2} = \frac{-\csc x}{1 + \csc x}$   
 g)  $f'(x) = \frac{-\csc x - \csc x \sec^2 x}{\tan^2 x}$     h)  $f'(x) = \frac{2x \cos x - 2x + x^2 \sin x + \sin x}{(\cos x - 1)^2}$   
 i)  $f'(x) = \frac{-3 \cos x}{(1 + 2 \sin x)^2}$     j)  $f'(x) = -\frac{\sin x}{2\sqrt{x}} + \sqrt{x} \cos x - \frac{\cos x}{2x^{\frac{3}{2}}}$   
 k)  $f'(\theta) = \frac{3}{(2 \cos \theta - \sin \theta)^2}$     l)  $f'(x) = \frac{1}{(1 + x \tan x)^2}$
7. a)  $y_T = \sqrt{3}x - \frac{\sqrt{3}\pi}{6} + 1$     b)  $y_T = 6x - \frac{9\pi}{2} - 3$     c)  $y_T = x - 1$   
 $y_N = -\frac{\sqrt{3}}{3}x + \frac{\sqrt{3}\pi}{18} + 1$      $y_N = \frac{-1}{6}x + \frac{\pi}{8} - 3$      $y_N = -x + 2\pi - 1$   
 d)  $y_T = 2\sqrt{2}$     e)  $y_T = 8x - \frac{20\pi}{3} + 2\sqrt{3}$   
 $x_N = \frac{\pi}{4}$      $y_N = \frac{-1}{8}x + \frac{5\pi}{48} + 2\sqrt{3}$
8.  $x = \frac{\pi}{6} \pm 2\pi n, \frac{5\pi}{6} \pm 2\pi n \quad n \in \mathbb{Z}$
9. a)  $v(t) = 8\cos t$     b) Position:  $4\sqrt{3}$  cm    Velocity:  $-4$  cm/s, moving towards the wall
10. 2.7 Newtons/degrees