



## MATHEMATICS 201-NYA-05

Differential Calculus

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# VI – Rules of Differentiation

1. Differentiate each function.

a)  $f(x) = 4x^3$

c)  $f(x) = \frac{3x^5}{10} - \sqrt{2}$

e)  $f(x) = x^{-3} + \frac{1}{x}$

g)  $f(t) = \frac{1}{3t^2} - \frac{1}{2t} + 3$

i)  $f(t) = \frac{1}{3}t^{-3} - 3t^3 + \frac{1}{\sqrt{t}}$

k)  $f(t) = \sqrt[3]{t^2} - \frac{1}{\sqrt{t^5}} + \sqrt[5]{t} + \sqrt{3}$

m)  $f(x) = (x^3 - x)(x^2 + 2x)$

o)  $f(x) = \left(\frac{1}{x^2} + x\right)\left(\frac{1}{x} - x^2\right)$

q)  $f(x) = \sqrt{x}(\sqrt[3]{x} + x + 1)$

s)  $f(x) = \frac{2x^2 + x + 1}{x^2 - 3x + 1}$

u)  $f(x) = \frac{-3}{x^2 + 2x - 5}$

w)  $f(x) = \frac{x^2 + 3x + 5}{\sqrt{x}}$

y)  $f(x) = \frac{ax + b}{cx + d}$

b)  $f(x) = \frac{3}{2}x^6 - 3x + 1$

d)  $f(x) = \frac{x^{10}}{2} - \frac{x^5}{5} + \frac{1}{4}$

f)  $f(x) = x^5 + x^{-5} + \frac{1}{x^3}$

h)  $f(x) = \sqrt{x} + \frac{3}{2x} - 5x^3$

j)  $f(x) = x^{\frac{2}{3}} - \frac{4}{x^{\frac{5}{7}}} + 3x^{\frac{1}{2}} - 4x^{\sqrt{2}}$

l)  $f(x) = x^2(2x^3 - 1)$

n)  $f(x) = \sqrt{x}(x^3 - 3\sqrt{x} + 1)$

p)  $f(x) = \frac{2x + 3}{4x - 5}$

r)  $f(u) = (u + 1)(u^3(u + 5))$

t)  $f(t) = \frac{t^3}{3t^4 + 1}$

v)  $f(x) = \left(\frac{3x - 3}{2x + 1}\right)(4x^2 + 5)$

x)  $f(x) = \frac{\sqrt{x} + 1}{\sqrt{x} - 1}$

z)  $f(x) = \frac{1 + \frac{4}{x}}{2 - \frac{5}{x+3}}$

2. Find  $f'(3)$ .

a)  $f(x) = \frac{1}{4}x^4$

c)  $f(x) = (x^3 + 1)(x - 4)$

b)  $f(x) = \frac{1}{x} + 2\sqrt{x} - 3$

d)  $f(x) = \frac{x^3 - 2}{x^2 + x}$



10. Find the points on the graph of  $f(x) = \frac{10}{x^2 - 12}$  where the tangent line passes through the origin.

11. How many tangent lines to the curve  $y = \frac{x}{x+1}$  pass through the point  $(1, 2)$ ? At which points do these tangent lines touch the curve?

12. Find the equation of all tangent lines to the curve  $f(x) = x^2 + 2x + 9$  that pass through the point  $(3, -1)$ .

13. Find all points on the graph of the function at which the tangent line is horizontal.

a) $f(x) = x^3 - 6x^2 + 9x + 4$	b) $f(x) = \frac{1}{x} + \frac{1}{x^2}$
c) $f(x) = 2\sqrt{x} - x + 1$	d) $f(x) = \frac{x^2}{x+1}$

14. The position function of a particle is given by

$$s = \frac{t}{t^2 + 1} \quad t \geq 0$$

where  $t$  is in seconds and  $s$  in meters.

- Find the velocity at time  $t$ .
- What is the velocity after 3 s?
- When is the particle at rest?

15. If a ball is thrown vertically upward with a velocity of 24.5 m/s, then its height after  $t$  seconds is  $s = 24.5t - 4.9t^2$ .

- What is the maximum height reached by the ball?
- What is the velocity of the ball when it is 29.4 m above the ground on its way up? On its way down?

16. A spherical balloon is being inflated. Find the rate of increase of the surface area ( $S = 4\pi r^2$ ) with respect to the radius  $r$  when  $r$  is (a) 1 m, (b) 2 m and (c) 3 m.

17. If a tank holds 5000 liters of water, which drains from the bottom of the tank in 40 minutes, then Torricelli's Law gives the volume  $V$  of water remaining in the tank after  $t$  minutes as

$$V = 5000\left(1 - \frac{t}{40}\right)^2 \quad 0 \leq t \leq 40$$

Find the rate at which water is draining from the tank after (a) 5 min, (b) 10 min, (c) 20 min and (d) 40 min. At what time is the water flowing out the fastest? The slowest?

18. Wildlife biologists predict that the population  $N$  of a certain endangered species after  $t$  years will be given by the equation  $N = (3t + 150)(50 - t)$  for  $0 \leq t \leq 50$  years. If this prediction is correct, find the rate of change 20 years from now. What does your answer represent?

## ANSWERS

1. a)  $f'(x) = 12x^2$       b)  $f'(x) = 9x^5 - 3$       c)  $f'(x) = \frac{3}{2}x^4$   
 d)  $f'(x) = 5x^9 - x^4$       e)  $f'(x) = \frac{-3}{x^4} - \frac{1}{x^2}$       f)  $f'(x) = 5x^4 - \frac{5}{x^6} - \frac{3}{x^4}$   
 g)  $f'(t) = \frac{-2}{3t^3} + \frac{1}{2t^2}$       h)  $f'(x) = \frac{1}{2\sqrt{x}} - \frac{3}{2x^2} - 15x^2$       i)  $f'(t) = \frac{-1}{t^4} - 9t^2 - \frac{1}{2t^{\frac{3}{2}}}$   
 j)  $f'(x) = \frac{2}{3x^{\frac{1}{3}}} + \frac{20}{7x^{\frac{12}{7}}} + \frac{3}{2\sqrt{x}} - 4\sqrt{2}x^{\sqrt{2}-1}$       k)  $f'(t) = \frac{2}{3t^{\frac{1}{3}}} + \frac{5}{2t^{\frac{7}{2}}} + \frac{1}{5t^{\frac{4}{5}}}$   
 l)  $f'(x) = 10x^4 - 2x$       m)  $f'(x) = 5x^4 + 8x^3 - 3x^2 - 4x$   
 n)  $f'(x) = \frac{7}{2}x^{\frac{5}{2}} - 3 + \frac{1}{2\sqrt{x}}$       o)  $f'(x) = \frac{-3 - 3x^6}{x^4}$       p)  $f'(x) = \frac{-22}{(4x - 5)^2}$   
 q)  $f'(x) = \frac{5}{6x^{\frac{1}{6}}} + \frac{3}{2}\sqrt{x} + \frac{1}{2\sqrt{x}}$       r)  $f'(u) = 5u^4 + 24u^3 + 15u^2$   
 s)  $f'(x) = \frac{4 + 2x - 7x^2}{(x^2 - 3x + 1)^2}$       t)  $f'(t) = \frac{-3t^6 + 3t^2}{(3t^4 + 1)^2}$       u)  $f'(x) = \frac{6x + 6}{(x^2 + 2x - 5)^2}$   
 v)  $f'(x) = \frac{48x^3 + 12x^2 - 24x + 45}{(2x + 1)^2}$       w)  $f'(x) = \frac{3x^2 + 3x - 5}{2x^{\frac{3}{2}}}$   
 x)  $f'(x) = \frac{-1}{\sqrt{x}(\sqrt{x} - 1)^2}$       y)  $f'(x) = \frac{ad - cb}{(cx + d)^2}$       z)  $f'(x) = \frac{-13x^2 - 48x - 12}{x^2(2x + 1)^2}$
2. a) 27      b)  $\frac{\sqrt{3}}{3} - \frac{1}{9}$       c) 1      d)  $\frac{149}{144}$       3. a) 8      b) -9      c) 22      d)  $\frac{1}{8}$
4. a)  $y_T = 6x - 3$       b)  $y_T = \frac{7}{8}x + \frac{3}{2}$       c)  $y_T = -2x + 7$       d)  $y_T = 8$   
 $y_N = \frac{-1}{6}x + \frac{28}{3}$        $y_N = \frac{-8}{7}x + \frac{67}{7}$        $y_N = \frac{1}{2}x + 2$        $x_N = 4$
5. a) -1      b) 1      6.  $(\frac{9}{2}, \frac{101}{4})$        $y = 9x - \frac{61}{4}$
7. a)  $y = 3x - \frac{13}{3}$       b)  $y = -x + 1$       c)  $y = \frac{1}{2}x + \frac{7}{2}$ ,  $y = \frac{1}{2}x - \frac{1}{2}$       d)  $y = \frac{1}{4}x + 1$   
 e)  $y = -2x$ ,  $y = -2x + 3\sqrt{3}$ ,  $y = -2x - 3\sqrt{3}$       f)  $y = -5x - \frac{19}{3}$ ,  $y = -5x + \frac{13}{3}$
8.  $a = \frac{-1}{2}$ ,  $b = 2$       9. (-1, 6) (5, 30)      10.  $(\pm 2, \frac{-5}{4})$
11. Two,  $P_1(-2 - \sqrt{3}, \frac{1}{2} + \frac{\sqrt{3}}{2})$   $P_2(-2 + \sqrt{3}, \frac{1}{2} - \frac{\sqrt{3}}{2})$       12.  $y = 18x - 55$ ,  $y = -2x + 5$
13. a) (1, 8) (3, 4)      b)  $(-2, \frac{-1}{4})$       c) (1, 2)      d) (0, 0) (-2, -4)
14. a)  $v(t) = \frac{1-t^2}{(t^2+1)^2}$       b)  $\frac{-2}{25}$  m/s      c)  $t = 1$  s
15. a) 30.625 m      b) 4.9 m/s - 4.9 m/s
16. a)  $8\pi$  m<sup>2</sup>/m      b)  $16\pi$  m<sup>2</sup>/m      c)  $24\pi$  m<sup>2</sup>/m
17. a) -218.75 l/min      b) -187.5 l/min      c) -125 l/min      d) 0 l/min  
 Fastest when  $t = 0$  min and slowest when  $t = 40$  min.
18. -120 The population is decreasing at a rate of 120 animals per year.