



## MATHEMATICS 201-NYA-05

Differential Calculus

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# V – Tangents and Derivatives

- Find the slope of the tangent line (using the definition with  $t$ ) for the curve  $y = f(x)$  at the given point.
  - $y = 5x - 3$  at  $x = -2$
  - $y = 3x^2 - 1$  at  $x = 1$
  - $y = 2x^2 - 4x + 5$  at  $x = -1$
  - $y = \sqrt{x+1}$  at  $x = 8$
  - $y = \frac{1}{2x-3}$  at  $x = 2$
- Find the equation of the tangent line for each of the curves in question 1 to the curve  $y = f(x)$  at the given point.
- Find the slope of the tangent line (using the definition with  $\Delta x$ ) for the curve  $y = f(x)$  at the given point and use your result to find the equation of the tangent line.
  - $y = 3x^2 - 2x + 1$  at  $x = 3$
  - $y = x - 5x^2$  at  $x = 1$
  - $y = x^3 + 2x + 1$  at  $x = -1$
  - $y = \sqrt{2-x} + 3$  at  $x = -2$
  - $y = \frac{x}{x+1}$  at  $x = 3$
- Find the derivative of the function by direct use of the definition.

a) $f(x) = 4x - 3$	b) $f(x) = 3x^2$
c) $f(x) = 2 - x^2$	d) $f(x) = 2x^2 - 3x + 4$
e) $f(x) = 5 - x - 3x^2$	f) $f(x) = 2x^3 - 1$
g) $f(x) = x^5$	h) $f(x) = \sqrt{x+1}$
i) $f(x) = 3 - 2\sqrt{5x+1}$	j) $f(x) = \sqrt{1-9x^2}$
k) $f(x) = \frac{2}{3+x}$	l) $f(x) = 3 - \frac{x+1}{x+4}$
m) $f(x) = \frac{1}{\sqrt{x+1}}$	n) $f(x) = \frac{1}{x^2+1}$

5. Find the derivative of the function by direct use of the alternate definition.

a)  $f(x) = 3 + 4x^2$

b)  $f(x) = 5x - x^2$

c)  $f(x) = x^3 - 8$

d)  $f(x) = 2 - \sqrt{x-3}$

e)  $f(x) = \frac{x}{x-3}$

f)  $f(x) = \frac{1}{x^2}$

6. For each of the functions  $f(x)$ ,

i) find  $f'(x)$

ii) the equation of the tangent line at  $x = a$  using your answer for (i).

a)  $f(x) = 2x^2 - 3$  at  $x = 3$

b)  $f(x) = x^3 + x^2$  at  $x = -1$

c)  $f(x) = \sqrt{1-x}$  at  $x = -3$

d)  $f(x) = \frac{1}{x} + x$  at  $x = 1$

7. At what point does the curve  $y = f(x)$  have a horizontal tangent?

a)  $f(x) = x^2 - x$

b)  $f(x) = 3x^3 - x$

c)  $f(x) = x^4$

8. The displacement (in meters) of a particle moving in a straight line is given by  $s = 3t^2 + t - 1$ , where  $t$  is measured in seconds.

a) Find the average velocity over the time interval  $[3, 4]$ .

b) Find the instantaneous velocity when  $t = 3$ .

9. The displacement (in meters) of a particle moving in a straight line is given by  $s = t^3 - 2t + 1$ , where  $t$  is measured in seconds.

a) Find the average velocity over the time interval  $[0, 2]$ .

b) Find the average velocity over the time interval  $[2, 4]$ .

c) Find the instantaneous velocity when  $t = 2$ .

10. If a ball is thrown into the air with a velocity of 10 m/s, its height (in meters) after  $t$  seconds is given by  $y = 10t - 4.9t^2$ .

a) Find the velocity after 2 seconds.

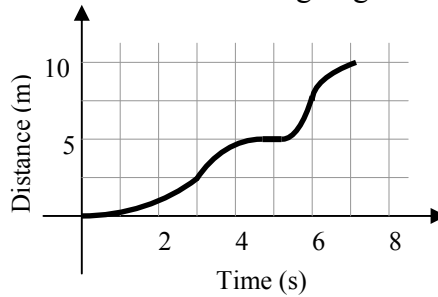
b) When will the ball reach its highest point?

c) When will the ball be back on the ground?

11. The temperature of a heated plate (in °C) is given by  $T = \frac{100}{x+1}$  where  $x$  is the distance from the center (in cm). Find the rate of change of the temperature with respect to the distance from the center 3 cm away from the center.

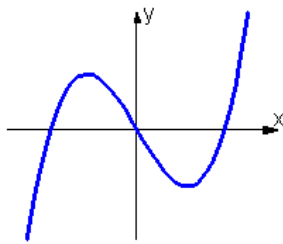
12. The accompanying figure shows the position versus time of car going in a straight line.

- a) When is the car at rest? (not moving)
- b) When is the car going the fastest?

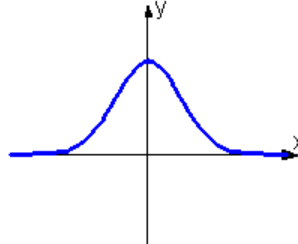


13. For each of the function  $f(x)$  whose graph is given below, sketch the graph of  $f'(x)$ .

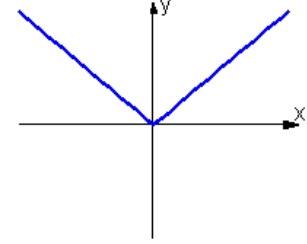
a)



b)

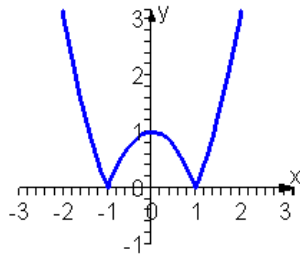


c)

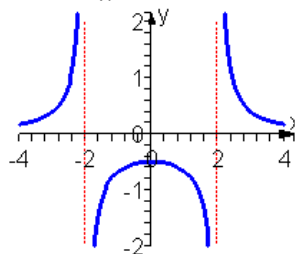


14. Determine the  $x$  values for which the function is not differentiable.

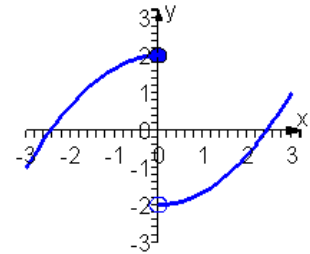
a)  $f(x) = |x^2 - 1|$



b)  $f(x) = \frac{2}{x^2 - 4}$



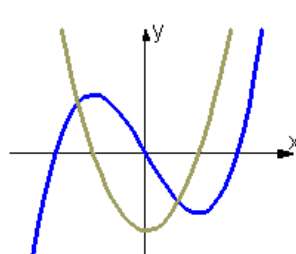
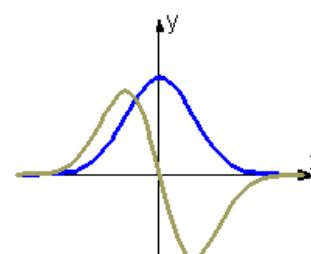
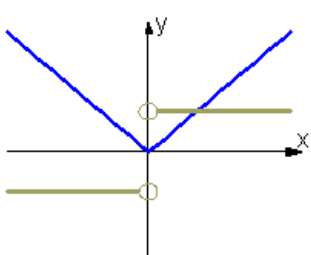
c)  $f(x) = \begin{cases} 2 - x^2 & x \leq 0 \\ x^2 - 2 & x > 0 \end{cases}$



15. Show that the function  $f(x) = |x - 4|$  is not differentiable at  $x = 4$ .

16. Show that the function  $f(x) = \begin{cases} x & x < 1 \\ 2 - 4x & x \geq 1 \end{cases}$  is not differentiable at  $x = 1$ .

## ANSWERS

1. a) 5      b) 6      c) -8      d)  $\frac{1}{6}$       e) -2
2. a)  $y = 5x - 3$       b)  $y = 6x - 4$       c)  $y = -8x + 3$       d)  $y = \frac{1}{6}x + \frac{5}{3}$       e)  $y = -2x + 5$
3. a)  $y = 16x - 26$       b)  $y = -9x + 5$       c)  $y = 5x + 3$       d)  $y = \frac{-1}{4}x + \frac{9}{2}$       e)  $y = \frac{1}{16}x + \frac{9}{16}$
4. a)  $f'(x) = 4$       b)  $f'(x) = 6x$       c)  $f'(x) = -2x$       d)  $f'(x) = 4x - 3$   
 e)  $f'(x) = -1 - 6x$       f)  $f'(x) = 6x^2$       g)  $f'(x) = 5x^4$       h)  $f'(x) = \frac{1}{2\sqrt{x+1}}$   
 i)  $f'(x) = \frac{-5}{\sqrt{5x+1}}$       j)  $f'(x) = \frac{-9x}{\sqrt{1-9x^2}}$       k)  $f'(x) = \frac{-2}{(3+x)^2}$       l)  $f'(x) = \frac{-3}{(x+4)^2}$   
 m)  $f'(x) = \frac{-1}{2(x+1)^{\frac{3}{2}}}$       n)  $f'(x) = \frac{-2x}{(x^2+1)^2}$
5. a)  $f'(x) = 8x$       b)  $f'(x) = 5 - 2x$       c)  $f'(x) = 3x^2$       d)  $f'(x) = \frac{-1}{2\sqrt{x-3}}$   
 e)  $f'(x) = \frac{-3}{(x-3)^2}$       f)  $f'(x) = \frac{-2}{x^3}$
6. a)  $f'(x) = 4x$        $y = 12x - 21$       b)  $f'(x) = 3x^2 + 2x$        $y = x + 1$   
 c)  $f'(x) = \frac{-1}{2\sqrt{1-x}}$        $y = \frac{-1}{4}x + \frac{5}{4}$       d)  $f'(x) = \frac{-1}{x^2} + 1$        $y = 2$
7. a)  $(\frac{1}{2}, \frac{-1}{4})$       b)  $(\frac{1}{3}, \frac{-2}{9})$        $(\frac{-1}{3}, \frac{2}{9})$       c)  $(0, 0)$
8. a) 22 m/s      b) 19 m/s
9. a) 2 m/s      b) 26 m/s      c) 10 m/s
10. a) -9.6 m/s      b)  $\approx 1.02$  s      c)  $\approx 2.04$  s
11.  $-\frac{25}{4}^\circ\text{C}/\text{cm}$
12. a)  $t = 0, t = 5$       b)  $t = 6$
13. a)       b)       c) 
14. a)  $x = \pm 1$       b)  $x = \pm 2$
15.  $f'_-(4) = -1$        $f'_+(4) = 1$        $\therefore f'(4) \nexists$
16.  $f'_-(1) = -\infty$        $f'_+(1) = -4$        $\therefore f'(1) \nexists$