



MATHEMATICS 201-NYA-05

Differential Calculus

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Rules of Differentiation

Definition

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Alternative Definition

$$f'(x) = \lim_{t \rightarrow x} \frac{f(t) - f(x)}{t - x}$$

Rules

1. $(cf)' = cf'$

3. $(f - g)' = f' - g'$

5. $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

7. $\frac{d}{dx}[c] = 0$

9. $\frac{d}{dx}[e^x] = e^x$

11. $\frac{d}{dx}[\ln x] = \frac{1}{x}$

13. $\frac{d}{dx}[\sin x] = \cos x$

14. $\frac{d}{dx}[\tan x] = \sec^2 x$

17. $\frac{d}{dx}[\sec x] = \sec x \tan x$

19. $\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$

21. $\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}$

23. $\frac{d}{dx}[\operatorname{arcsec} x] = \frac{1}{x\sqrt{x^2-1}}$

2. $(f + g)' = f' + g'$

4. $(fg)' = f'g + fg'$

6. $\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$ or $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

8. $\frac{d}{dx}[x^n] = nx^{n-1}$

10. $\frac{d}{dx}[a^x] = a^x \ln a$

12. $\frac{d}{dx}[\log_a x] = \frac{1}{x \ln a}$

14. $\frac{d}{dx}[\cos x] = -\sin x$

16. $\frac{d}{dx}[\cot x] = -\operatorname{csc}^2 x$

18. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

20. $\frac{d}{dx}[\arccos x] = -\frac{1}{\sqrt{1-x^2}}$

22. $\frac{d}{dx}[\operatorname{arccot} x] = -\frac{1}{1+x^2}$

24. $\frac{d}{dx}[\operatorname{arccsc} x] = -\frac{1}{x\sqrt{x^2-1}}$

Parametric Equations

$$\begin{cases} x = f(t) \\ y = g(t) \end{cases} \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{d^2 y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$