



QUIZ #4 B

Solutions

*Answer all questions and show all your work. Exact answers are required.
No calculators are permitted.*

Question 1 (6 points)

Find the limit.

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\tan 6x}{5x} &= \lim_{x \rightarrow 0} \frac{\frac{\sin 6x}{\cos 6x}}{5x} = \lim_{x \rightarrow 0} \frac{\sin 6x}{6x} \frac{6}{5 \cos 6x} \\ &= \lim_{x \rightarrow 0} \frac{\sin 6x}{6x} \lim_{x \rightarrow 0} \frac{6}{5 \cos 6x} \\ &= 1 \cdot \frac{6}{5} = \frac{6}{5}\end{aligned}$$

Question 2 (6 points)

Find $f'(x)$ if $f(x) = \frac{\tan x}{2 + \sec x}$

$$\begin{aligned}f'(x) &= \frac{\frac{d}{dx}[\tan x](2 + \sec x) - \tan x \frac{d}{dx}[2 + \sec x]}{(2 + \sec x)^2} \\ &= \frac{\sec^2 x(2 + \sec x) - \sec x \sec x \tan x}{(2 + \sec x)^2} \\ &= \frac{2\sec^2 x + \sec^3 x - \sec^2 x \tan x}{(2 + \sec x)^2}\end{aligned}$$

Question 3 (4 points)

Find the equation of the tangent line to the graph of $f(x) = \sin x \cos x$ at $x = \frac{\pi}{3}$.

$$\begin{aligned}f'(x) &= \frac{d}{dx}[\sin x] \cos x + \sin x \frac{d}{dx}[\cos x] \\ &= \cos^2 x - \sin^2 x\end{aligned}$$

$$m = f'\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{-1}{2}$$

$$f\left(\frac{\pi}{3}\right) = \frac{1}{2} \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$

$$y = mx + b$$

$$\frac{\sqrt{3}}{4} = \frac{-1}{2} \frac{\pi}{3} + b$$

$$b = \frac{\sqrt{3}}{4} + \frac{\pi}{6}$$

$$\text{Hence } y = \frac{-1}{2}x + \frac{\sqrt{3}}{4} + \frac{\pi}{6}$$