



MATHEMATICS 201-NYA-05

Differential Calculus

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Precalculus - Functions

1. Find the domain for the following functions.

a) $f(x) = \sqrt{x-3}$

b) $f(x) = \frac{x-2}{x^2-4}$

c) $f(x) = x - x^{-1}$

d) $f(x) = x^2 - \sqrt{x} + 2$

e) $f(x) = \frac{x}{x^2+1}$

f) $f(x) = 1 - \sqrt[3]{x+2}$

2. Let $f(x) = 2x^2 - 4x + 2$. Find

a) $f(-1)$

b) $f(\sqrt{2})$

c) $f(\sqrt{2}+3)$

d) $f(t^2)$

e) $f(2x)$

f) $f(3x-4)$

g) $\frac{f(x+h)-f(x)}{h}$

h) $\frac{f(t)-f(x)}{t-x}$

3. Let $f(x) = \frac{2-x}{2+x}$. Find

a) $f(5)$

b) $f(-2)$

c) $f(\frac{1}{2})$

d) $f(x+3)$

e) $f(\frac{1}{x})$

f) $f(\frac{x}{x-1})$

g) $\frac{f(x+h)-f(x)}{h}$

h) $\frac{f(t)-f(x)}{t-x}$

4. For each of the following functions, find $\frac{f(x+h)-f(x)}{h}$.

a) $f(x) = x^2 - 5x + 6$

b) $f(x) = x^3 - x$

c) $f(x) = \frac{1}{x^2}$

d) $f(x) = \frac{4}{2x+1}$

e) $f(x) = x + \frac{1}{x}$

f) $f(x) = \frac{x}{x^2+4}$

5. Let $f(x) = 2x^2 - x + 1$, $g(x) = \frac{1}{x-1}$ and $h(x) = 2x + \frac{3}{x}$. Find the following functions.

a) $f+g$

b) $f-g$

c) fg

d) $\frac{f}{g}$

e) $f \circ g$

f) $g \circ f$

g) $f \circ f$

h) $g \circ g$

i) $f \circ h$

j) $h \circ g$

k) $h \circ (g \circ f)$

6. Find an equation of the line that satisfies the given conditions.

- Passing through $P(2,-3)$ and $Q(4,5)$.
- Passing through $P(1,2)$ and $Q(5,2)$.
- Passing through $P(1,2)$ and $Q(1,-8)$
- x -intercept 1 and y -intercept -3
- Passing through $P(3,-4)$ and parallel to $x + 3y = 6$.
- Passing through $P(3,-4)$ and perpendicular to $3x - 9y = 1$

7. For each of the following functions,

- find the domain,
- find the intercepts,
- sketch the graph,
- find the range.

- | | | |
|--------------------------|------------------------------|----------------------------|
| a) $f(x) = 2x - 5$ | b) $f(x) = -3x + 6$ | c) $f(x) = x^2 - 9$ |
| d) $f(x) = (x + 2)^2$ | e) $f(x) = -(x - 2)^2 + 4$ | f) $f(x) = 2(x + 1)^2 + 3$ |
| g) $f(x) = x^2 - 2x + 1$ | h) $f(x) = x^2 - 4x - 5$ | i) $f(x) = x - x^2$ |
| j) $f(x) = \sqrt{x + 2}$ | k) $f(x) = \sqrt{x - 2} - 3$ | l) $y = x^6 - 1$ |
| m) $y = x^3 + 3$ | n) $f(x) = x + 1 $ | o) $f(x) = x^2 - 1 $ |

8. Sketch the graph of the following functions.

- | | |
|---|---|
| a) $f(x) = \begin{cases} x & x < 1 \\ x^2 & x \geq 1 \end{cases}$ | b) $f(x) = \begin{cases} 4 - x^2 & x < -1 \\ x^2 + 4 & x \geq -1 \end{cases}$ |
| c) $f(x) = \begin{cases} \frac{1}{x} & x < 0 \\ 3 & x = 0 \\ x^2 & x > 0 \end{cases}$ | d) $f(x) = \begin{cases} x - 2 & x \neq -2 \\ 1 & x = -2 \end{cases}$ |
| e) $f(x) = \begin{cases} 3 - x & x \leq -2 \\ x^3 - 1 & -2 < x \leq 1 \\ x^2 - x & x > 1 \end{cases}$ | f) $f(x) = -\lceil 1 - x \rceil$ |

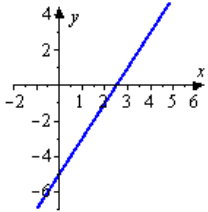
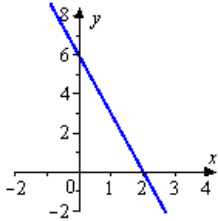
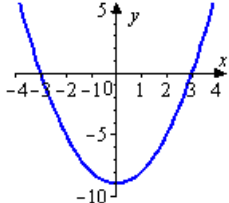
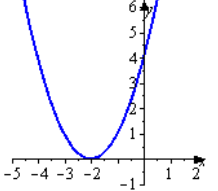
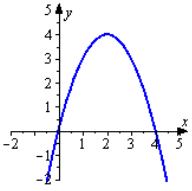
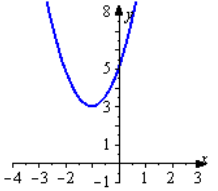
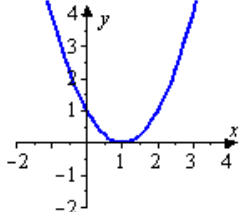
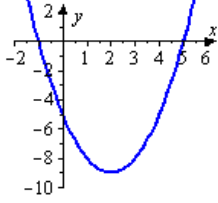
9. The Heaviside function H is defined by

$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$

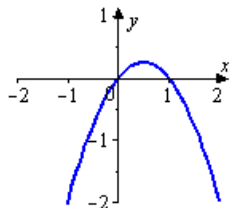
It is used in the study of electric circuits to represent the sudden surge of electric current, or voltage, when a switch is instantaneously turned on.

- Sketch the graph of the Heaviside function.
- Write a formula for the voltage $V(t)$ in a circuit if the switch is turned on at time $t = 0$ and 120 volts are applied instantaneously to the switch.
- Write a formula for the voltage $V(t)$ in a circuit if the switch is turned on at time $t = 5$ and 240 volts are applied instantaneously to the switch.

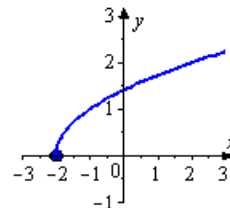
ANSWERS

1. a) $[3, \infty)$ b) $\mathbb{R} / \{\pm 2\}$ c) $\mathbb{R} / \{0\}$ d) $[0, \infty)$ e) \mathbb{R} f) \mathbb{R}
2. a) 8 b) $6 - 4\sqrt{2}$ c) $12 + 8\sqrt{2}$ d) $2t^4 - 4t^2 + 2$
 e) $8x^2 - 8x + 2$ f) $18x^2 - 60x + 50$ g) $4x + 2h - 4$ h) $2t + 2x - 4$
3. a) $\frac{3}{7}$ b) $\cancel{4}$ c) $\frac{3}{5}$ d) $-\frac{x+1}{x+5}$
 e) $\frac{2x-1}{2x+1}$ f) $\frac{x-2}{3x-2}$ g) $\frac{-4}{(2+x)(2+x+h)}$ h) $\frac{-4}{(t+2)(x+2)}$
4. a) $2x + h - 5$ b) $3x^2 + 3xh + h^2 - 1$ c) $-\frac{2x+h}{(x+h)^2 x^2}$
 d) $\frac{-8}{(2x+2h+1)(2x+1)}$ e) $\frac{x^2+xh-1}{x(x+h)}$ f) $-\frac{x^2+xh-4}{(x^2+2xh+h^2+4)(x^2+4)}$
5. a) $(f+g)(x) = 2x^2 - x + 1 + \frac{1}{x-1}$ b) $(f-g)(x) = 2x^2 - x + 1 - \frac{1}{x-1}$
 c) $(fg)(x) = \frac{2x^2-x+1}{x-1}$ d) $(\frac{f}{g})(x) = 2x^3 - 3x^2 + 2x - 1$ e) $(f \circ g)(x) = \frac{x^2-3x+4}{(x-1)^2}$
 f) $(g \circ f)(x) = \frac{1}{2x^2-x}$ g) $(f \circ f)(x) = 8x^4 - 8x^3 + 8x^2 - 3x + 2$
 h) $(g \circ g)(x) = \frac{1-x}{x-2}$ i) $(f \circ h)(x) = 8x^2 - 2x + 25 - \frac{3}{x} + \frac{18}{x^2}$
 j) $(h \circ g)(x) = \frac{3x^2-6x+5}{x-1}$ k) $(h \circ (g \circ f))(x) = \frac{12x^4-12x^3+3x^2+2}{2x^2-x}$
6. a) $y = 4x - 11$ b) $y = 2$ c) $x = 1$ d) $y = 3x - 3$ e) $y = \frac{1}{3}x - 3$ f) $y = -3x + 5$
7. a) D : \mathbb{R}
 x-int: $x = \frac{5}{2}$
 y-int : $y = -5$
 R : \mathbb{R}
- 
- b) D : \mathbb{R}
 x-int: $x = 2$
 y-int : $y = 6$
 R : \mathbb{R}
- 
- c) D : \mathbb{R}
 x-int: $x = \pm 3$
 y-int : $y = -9$
 R : $[-9, \infty)$
- 
- d) D : \mathbb{R}
 x-int: $x = -2$
 y-int : $y = 4$
 R : $[0, \infty)$
- 
- e) D : \mathbb{R}
 x-int: $x = 0, 4$
 y-int : $y = 0$
 R : $(-\infty, 4]$
- 
- f) D : \mathbb{R}
 x-int: none
 y-int : $y = 5$
 R : $[3, \infty)$
- 
- g) D : \mathbb{R}
 x-int: $x = 1$
 y-int : $y = 1$
 R : $[0, \infty)$
- 
- h) D : \mathbb{R}
 x-int: $x = -1, 5$
 y-int : $y = -5$
 R : $[-9, \infty)$
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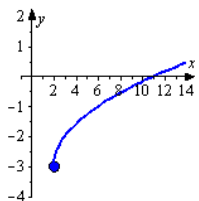
- i) $D : \mathbb{R}$
 x -int: $x = 0, 1$
 y -int : $y = 0$
 $R : (-\infty, \frac{1}{4}]$



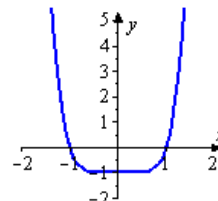
- j) $D : [-2, \infty)$
 x -int: $x = -2$
 y -int : $y = \sqrt{2}$
 $R : [0, \infty)$



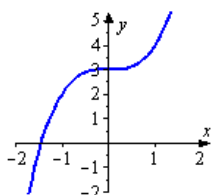
- k) $D : [2, \infty)$
 x -int: $x = 11$
 y -int : none
 $R : [-3, \infty)$



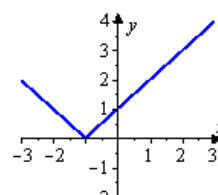
- l) $D : \mathbb{R}$
 x -int: $x = \pm 1$
 y -int : $y = -1$
 $R : [-1, \infty)$



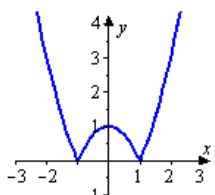
- m) $D : \mathbb{R}$
 x -int: $x = -\sqrt[3]{3}$
 y -int : $y = 3$
 $R : \mathbb{R}$



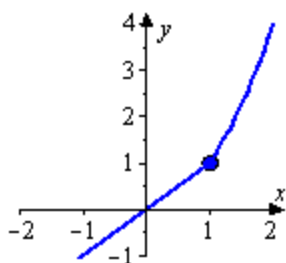
- n) $D : \mathbb{R}$
 x -int: $x = -1$
 y -int : $y = 1$
 $R : [0, \infty)$



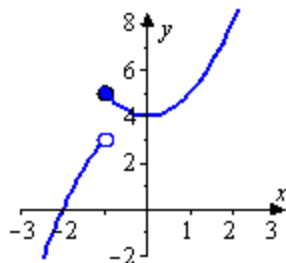
- o) $D : \mathbb{R}$
 x -int: $x = \pm 1$
 y -int : $y = 1$
 $R : [0, \infty)$



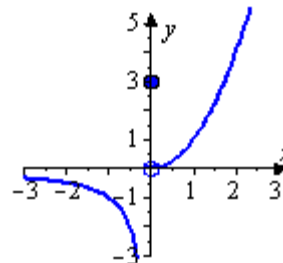
8. a)



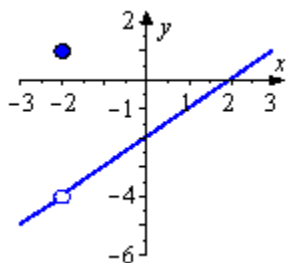
b)



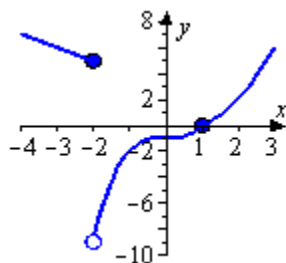
c)



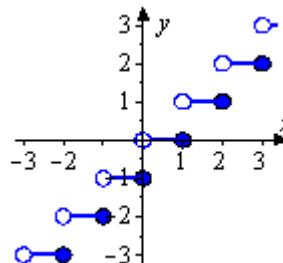
d)



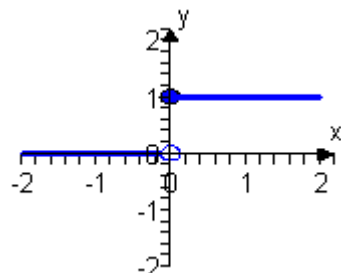
e)



f)



9. a)



b) $V(t) = 120H(t)$

c) $V(t) = 240H(t-5)$