CEGEP CHAMPLAIN ST. LAWRENCE
CHAMPLAIN REGIONAL COLLEGE

MATHEMATICS 201-NYA-05
Differential Calculus
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IX – Implicit Differentiation

1. Find $\dfrac{dy}{dx}$.

   a) $4x^2 + 9y^2 = 36$ b) $xy^2 + x^3 + y^3 = 4$ c) $\dfrac{1}{x} + \dfrac{1}{y} = 1$

   d) $x^4 y + \sqrt{xy} = 2$ e) $x^2 = \dfrac{x+y}{x-y}$ f) $\sqrt{x+y} + \sqrt{x-y} = 6$

   g) $\left(5x^2 y + 4\right)^7 = x^3$ h) $\dfrac{x^2}{y} - y = \dfrac{x}{2} - \dfrac{4}{y^2}$ i) $\sin^2 x + \cos^2 y = 1$

   j) $\sin\left(x^2 y^2\right) = x$ k) $\tan^3\left(xy^2 + y\right) = x$ l) $\cot\left(x + 2y\right) = 4xy$

2. Find an equation for the tangent line to the graph of the given curve at the specified point.

   a) $y^3 + yx + x^2 - 3y^2 = 0 \quad (0,3)$ b) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4 \quad \left(-1, 3\sqrt{3}\right)$

3. Find $\dfrac{da}{dt}$ by implicit differentiation.

   a) $a^2 t - 2t = a^3\sqrt{t}$ b) $t = \sin a$

4. Find $\dfrac{du}{dv}$.

   a) $u^3 v = \sin u - \tan v + 2$ b) $\left(u + 3v\right)^4 = u^2 v^3$

5. Show that the equation for the tangent line to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ at the point $\left(x_0, y_0\right)$ is given by $\dfrac{x_0 x}{a^2} + \dfrac{y_0 y}{b^2} = 1$.

6. Show that the normal line to the circle $x^2 + y^2 = r^2$ at $\left(x_0, y_0\right)$ passes through the origin.

7. Find the values of a and b for the curve $x^2 y^3 - 4x^3 + ay^4 = b$ if the point $(2,1)$ is on the curve and the tangent line at that point is $x - 5y + 3 = 0$.

8. Find all points on the given curve where there is a horizontal tangent, and where there is a vertical tangent.

   a) $x^2 + xy + y^2 = 12$ b) $\left(x^2 + y^2\right)^2 = 8\left(x^2 - y^2\right)$

ANSWERS

1. a) $\frac{dy}{dx}=\frac{-4x}{9y}$ b) $\frac{dy}{dx}=-\frac{y^2+3x^2}{2xy+3y^2}$

   c) $\frac{dy}{dx}=\frac{-y^2}{x^2}$ d) $\frac{dy}{dx}=-\frac{8x^3y\sqrt{xy}+y}{2x^4\sqrt{xy}+x}$

   e) $\frac{dy}{dx}=\frac{x^3-2x^2y+xy^2+y}{x}$ f) $\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}$

   g) $\frac{dy}{dx}=\frac{3x-70y\left(5x^2y+4\right)^6}{35x\left(5x^2y+4\right)^6}$ h) $\frac{dy}{dx}=\frac{4y^2x-y^3}{2x^2y+2y^3+16}$

   i) $\frac{dy}{dx}=\frac{\sin x\cos x}{\sin y\cos y}$ j) $\frac{dy}{dx}=\frac{1-2xy^2\cos\left(x^2y^2\right)}{2x^2y\cos\left(x^2y^2\right)}$

   k) $\frac{dy}{dx}=\frac{1-3y^2\tan^2\left(xy^2+y\right)\sec^2\left(xy^2+y\right)}{3(2xy+1)\tan^2\left(xy^2+y\right)\sec^2\left(xy^2+y\right)}$

   l) $\frac{dy}{dx}=-\frac{4y+\csc^2(x+2y)}{4x+2\csc^2(x+2y)}$

2. a) $y=\frac{-1}{3}x+3$ b) $y=\sqrt{3}x+4\sqrt{3}$

3. a) $\frac{da}{dt}=\frac{a^3+4\sqrt{t}-2a^2\sqrt{t}}{4at\sqrt{t}-6a^2t}$ b) $\frac{da}{dt}=\sec a$

4. a) $\frac{du}{dv}=\frac{u^3+\sec^2 v}{\cos u-3u^2v}$ b) $\frac{du}{dv}=\frac{3u^2v^2-12(u+3v)^3}{4(u+3v)^3-2uv^3}$

5. $m=\left.\frac{dy}{dx}\right|_{(x_0,y_0)}=-\frac{b^2x_0}{a^2y_0}$ $\quad y_0=-\frac{b^2x_0}{a^2y_0}x_0+B$ $\quad y=-\frac{b^2x_0}{a^2y_0}x+y_0-\frac{b^2x_0^2}{a^2y_0}$

   $B=y_0-\frac{b^2x_0^2}{a^2y_0}$ $\quad \frac{y_0y}{b^2}+\frac{x_0x}{a^2}=\frac{y_0^2}{b^2}+\frac{x_0^2}{a^2}=1$

6. $m_{\text{tangent}}=\left.\frac{dy}{dx}\right|_{(x_0,y_0)}=-\frac{x_0}{y_0}$ $\quad m_{\text{normal}}=\frac{y_0}{x_0}$ $\quad y_0=\frac{y_0}{x_0}x_0+B$ $\quad \therefore y=\frac{y_0}{x_0}x$

   $B=0$

   Thus the normal passes through the origin.

7. $a=52$ and $b=24$

8. a) Tangent: $(2,-4)$, $(-2,4)$ Vertical: $(4,-2)$, $(-4,2)$

   b) Tangent: $\left(\sqrt{3},1\right)$, $\left(\sqrt{3},-1\right)$, $\left(-\sqrt{3},1\right)$, $\left(-\sqrt{3},-1\right)$ Vertical: $\left(2\sqrt{2},0\right)$, $\left(-2\sqrt{2},0\right)$