CEGEP CHAMPLAIN ST. LAWRENCE CHAMPLAIN REGIONAL COLLEGE

MATHEMATICS 201-NYA-05
Differential Calculus
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IV - Continuity

1. Examine the continuity of f at $x=a$. If f is discontinuous at $x=a$, state the kind of discontinuity.

a) $f(x)=\begin{cases}\dfrac{x^2-x-6}{x-3} & x\neq 3\\ 5 & x=3\end{cases}$ at $x=3$

b) $f(x)=\begin{cases}\dfrac{x^2+x-6}{x-3} & x\neq 3\\ 5 & x=3\end{cases}$ at $x=3$

c) $f(x)=\begin{cases}\dfrac{x^2-x-6}{x-3} & x\neq 3\\ 3 & x=3\end{cases}$ at $x=3$

d) $f(x)=\begin{cases}x^2+1 & x\leq 2\\ 3x-4 & x>2\end{cases}$ at $x=2$

e) $f(x)=\begin{cases}4+x & x<1\\ x^2+4 & x\geq 1\end{cases}$ at $x=1$

f) $f(x)=\begin{cases}\dfrac{|x-3|}{x-3} & x<3\\ \dfrac{1}{x-4} & x\geq 3\end{cases}$ at $x=3$

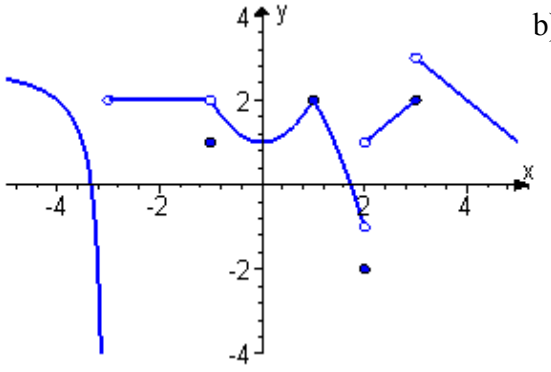
g) $f(x)=\begin{cases}3+x^2 & x<2\\ 0 & x=2\\ 11-x^2 & x>2\end{cases}$ at $x=2$

h) $f(x)=\begin{cases}[\![x]\!] & x\leq 4\\ \dfrac{x^2-5x+4}{x-4} & x>4\end{cases}$ at $x=4$

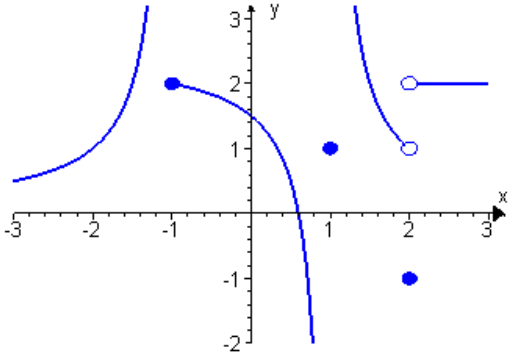
i) $f(x)=[\![1-x]\!]+[\![x-1]\!]$ at $x=1$

2. For each of the functions f whose graph is given below, find the points where f is discontinuous and state the type of discontinuity and find the intervals on which f is continuous.

a)



b)



3. Discuss the continuity of the following functions. If the function is discontinuous at a point, state the kind of discontinuity.

a) $f(x)=\dfrac{x^2+x-6}{x^2-4}$

b) $f(x)=\dfrac{1}{x}+\dfrac{1}{x-1}+\dfrac{1}{x^2-1}$

a) $f(x)=\sqrt{\dfrac{1}{x^2+1}}$

d) $f(x)=\left|x^2+2x+1\right|$

e) $f(x)=\begin{cases}\dfrac{x^2-x-6}{x-3} & x\neq 3\\ 3 & x=3\end{cases}$

f) $f(x)=\begin{cases}\dfrac{x}{x^2+1} & x\leq 1\\ \dfrac{1}{x-5} & x>1\end{cases}$

g) $f(x)=\begin{cases}x^2-1 & x<3\\ 2x+2 & x\geq 3\end{cases}$

h) $f(x)=\begin{cases}3x+1 & x<-2\\ \dfrac{x^2-1}{x} & x\geq -2\end{cases}$

i) $f(x)=\begin{cases}\dfrac{x^2+4}{x^2+2} & x\leq 0\\ \dfrac{x^2-4}{x^2-2x} & x>0\end{cases}$

j) $f(x)=\begin{cases}\dfrac{1}{x+2} & x\leq 1\\ \dfrac{\sqrt{x}-2}{x-4} & x>1\end{cases}$

k) $f(x)=\begin{cases}\dfrac{x}{x-1} & x\leq -1\\ \sqrt{x+1} & -1<x<4\\ \dfrac{2x-10}{x^2-7x+10} & x\geq 4\end{cases}$

l) $f(x)=\begin{cases}\dfrac{1}{x^2+2x} & x\leq -1\\ \dfrac{2}{x^2-x-2} & -1<x\leq 1\\ \dfrac{x^2-3x+2}{x-1} & x>1\end{cases}$

m) $f(x)=\begin{cases}\dfrac{x^2-16}{x^2+4x} & x<0\\ \dfrac{\sqrt{x+1}-2}{x^2-9} & 0\leq x\leq 3\\ \dfrac{1-\frac{3}{x}}{x^2+2x-15} & x>3\end{cases}$

4. Find all values of *a* and *b* that make $f(x)$ continuous on $\mathbb{R}$, where

$$f(x)=\begin{cases}3x-a & x\leq -1\\ x^2+1 & -1<x<2\\ b & x\geq 2\end{cases}$$

ANSWERS

1. a) Continuous b) Infinite discontinuity c) Removable discontinuity
   d) Jump discontinuity e) Continuous f) Continuous
   g) Removable Discontinuity h) Removable discontinuity i) Removable Discontinuity
2. a) Infinite discontinuity at $x=-3$ Removable discontinuity at $x=-1$
   Jump discontinuities at $x=2$ and $x=3$
   Continuous on $(-\infty,-3),(-3,-1),(-1,2),(2,3],(3,\infty)$
   b) Infinite discontinuities at $x=-1$ and $x=1$ Jump discontinuity at $x=2$
   Continuous on $(-\infty,-1),[-1,1),(1,2),(2,\infty)$
3. a) f is continuous on $(-\infty,-2)$, $(-2,2)$ and $(2,\infty)$ with a removable discontinuity at $x=2$ and an infinity discontinuity at $x=-2$.
   b) f is continuous on $(-\infty,-1)$, $(-1,0)$, $(0,1)$ and $(1,\infty)$ with infinity discontinuities at $x=0,\pm 1$.
   c) f is continuous on $\mathbb{R}$
   d) f is continuous on $\mathbb{R}$
   e) f is continuous on $(-\infty,3)$ and $(3,\infty)$ with a removable discontinuity at $x=3$
   f) f is continuous on $(-\infty,1]$, $(1,5)$ and $(5,\infty)$ with a jump discontinuity at $x=1$ and an infinite discontinuity at $x=5$
   g) f is continuous on $\mathbb{R}$
   h) f is continuous on $(-\infty,-2),[-2,0),(0,\infty)$ with a jump discontinuity at $x=-2$ and an infinite discontinuity at $x=0$
   i) f is continuous on $(-\infty,0],(0,2),(2,\infty)$ with an infinite discontinuity at $x=0$ and a removable discontinuity at $x=2$.
   j) f is continuous on $(-\infty,-2),(-2,4),(4,\infty)$ with an infinite discontinuity at $x=-2$ and a removable discontinuity at $x=4$.
   k) f is continuous on $(-\infty,-1]$, $(-1,4)$, $[4,5)$ and $(5,\infty)$ with jump discontinuities at $x=-1,4$ and a removable discontinuity at $x=5$.
   l) f is continuous on $(-\infty,-2)$, $(-2,-1]$ and $(-1,\infty)$ with infinite discontinuities at $x=-2,-1$
   m) f is continuous on $(-\infty,-4)$, $(-4,0)$, $[0,3)$ and $(3,\infty)$ with a removable discontinuities at $x=-4$ and $x=3$, and an infinite discontinuity at $x=0$.
4. $a=-5$ and $b=5$