MATHEMATICS 201-NYA-05
Differential Calculus
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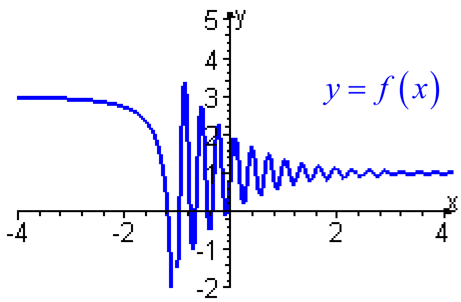
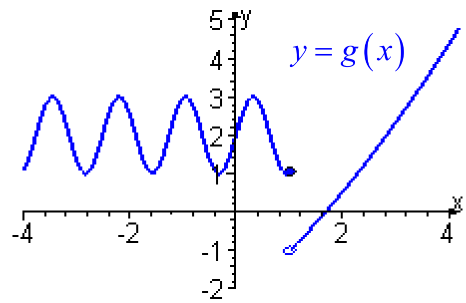
III – Limits at Infinity

1. Consider the functions f and g whose graphs are given below.





Find the following.

a) $\lim_{x\to\infty} f(x)$

b) $\lim_{x\to-\infty} f(x)$

c) $\lim_{x\to\infty} g(x)$

d) $\lim_{x\to-\infty} g(x)$

e) The equation of the horizontal asymptotes for f.

f) The equation of the horizontal asymptotes for g.

2. Evaluate the limit

a) $\lim_{x\to\infty} \frac{x+3}{2x-1}$

b) $\lim_{x\to\infty} \frac{x^3-3x+1}{1-4x^3}$

c) $\lim_{x\to-\infty} \frac{x^4-3x^2+1}{2x^4+x}$

d) $\lim_{x\to-\infty} \frac{3-x^3}{x^2+1}$

e) $\lim_{x\to\infty} \frac{x^2-3}{3-4x}$

f) $\lim_{x\to\infty} \frac{(x+3)^4(2-x)^3}{(x^3-3)^2(x+2)}$

g) $\lim_{x\to\infty} \frac{\sqrt{1+2x^2}}{2x}$

h) $\lim_{x\to-\infty} \frac{\sqrt{1+2x^2}}{2x}$

i) $\lim_{x\to\infty} \frac{1-\sqrt{x}}{1+\sqrt{x}}$

j) $\lim_{x\to\infty} \frac{6x}{\sqrt[4]{2x^4+1}}$

k) $\lim_{x\to-\infty} \frac{6x^2}{\sqrt[3]{8x^6+2}}$

l) $\lim_{x\to-\infty} \frac{\sqrt{x^4-x+1}}{x^2-3x}$

m) $\lim_{x\to\infty} \left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)$

n) $\lim_{x\to-\infty} \left(x+\sqrt{x^2+3x}\right)$

o) $\lim_{x\to\infty} \frac{2-x}{\sqrt{9+6x^2}}$

p) $\lim_{x\to-\infty} \frac{2-x}{\sqrt{9+6x^2}}$

q) $\lim_{x\to\infty} \left(x^3-3x+2\right)$

r) $\lim_{x\to-\infty} \left(2-3x-x^4\right)$

s) $\lim_{x\to-\infty} \left(2x^7-3x+1\right)$

t) $\lim_{x\to-\infty} \sqrt{4-3x}$

u) $\lim_{x\to\infty} \frac{1-\sqrt[5]{x}}{1+\sqrt[4]{x}}$

3. Find all horizontal asymptotes (if any) for the following functions.
   a) $f(x)=\dfrac{x^2+4}{x^2-4}$ b) $f(x)=\dfrac{2x^2-3}{x^3-27}$ c) $f(x)=3x^4-x+1$
   d) $f(x)=\dfrac{2\sqrt{x^2+4}}{x-3}$ e) $f(x)=\dfrac{1-3x^2}{\sqrt{x^4-3x}}$ f) $f(x)=\sqrt{x^2+3}-x$

4. Parks Canada introduced 30 elk into a new federal park. The population N of the herd is modeled by
$$N=\frac{10(3+4t)}{1+0.1t}$$
where t is in years.
   a) Find the size of the herd after 5, 10 and 25 years.
   b) According to this model, what is the limiting size of the herd as time progresses?

5. A tank contains 5000 L of pure water. Brine that contains 30 g of salt per liter of water is pumped into the tank at a rate of 25 L/min.
   a) Show that the concentration of salt, after t minutes (in grams per liter) is given by
$$C(t)=\frac{30t}{200+t}$$
   b) What happens to the concentration as $t\to\infty$?

ANSWERS

1. a) 1 b) 3 c) ∞ d) $\nexists$ e) $y=1,\quad y=3$ f) None
2. a) $\frac{1}{2}$ b) $\frac{-1}{4}$ c) $\frac{1}{2}$ d) ∞ e) $-\infty$ f) -1 g) $\frac{\sqrt{2}}{2}$
   h) $\frac{-\sqrt{2}}{2}$ i) -1 j) $3\sqrt[4]{8}$ k) 3 l) 1 m) 0 n) $\frac{-3}{2}$
   o) $\frac{-\sqrt{6}}{6}$ p) $\frac{\sqrt{6}}{6}$ q) ∞ r) $-\infty$ s) $-\infty$ t) ∞ u) 0
3. a) $y=1$ b) $y=0$ c) None
   d) $y=2,\quad y=-2$ e) $y=-3$ f) $y=0$
4. a) 153, 215 and 294 b) 400 elks
5. a) $C(t)=\dfrac{\text{amount of salt at time } t}{\text{total volume at time } t}=\dfrac{30\cdot 25t}{5000+25t}=\dfrac{30t}{200+t}$ b) $C(t)\to 30$ g/L