

MATHEMATICS 201-BNX-05

Introduction to College Math

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Test #3

Solutions

Question 1 (20 points)

Find the *exact* value of the following.

a) $\cos\left(-\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

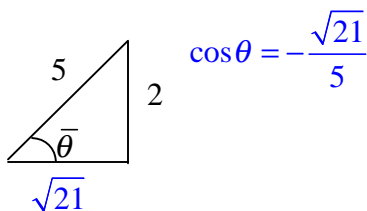
b) $\cot(120^\circ) = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{3}$

c) $\sec\left(-\frac{13\pi}{4}\right) = \frac{1}{-\frac{\sqrt{2}}{2}} = -\sqrt{2}$

d) $\sin(105^\circ) = \sin(60^\circ + 45^\circ)$
 $= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$
 $= \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} + \frac{1}{2} \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$

Question 2 (21 points)

If $\sin \theta = -\frac{2}{5}$ and θ is in quadrant III, find the *exact* value of



a) $\tan \theta = \frac{2}{\sqrt{21}} = \frac{2\sqrt{21}}{21}$

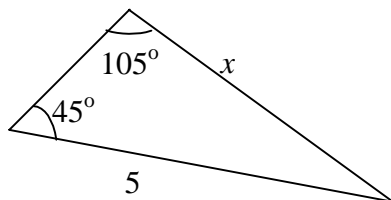
b) $\cos \frac{\theta}{2} = -\sqrt{\frac{1 + \cos \theta}{2}} = -\sqrt{\frac{1 - \frac{\sqrt{21}}{5}}{2}} = -\sqrt{\frac{5 - \sqrt{21}}{10}}$

c) $\sin 2\theta = 2 \sin \theta \cos \theta = 2\left(-\frac{2}{5}\right)\left(-\frac{\sqrt{21}}{5}\right) = \frac{4\sqrt{21}}{25}$

Question 3 (20 points)

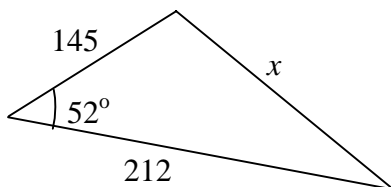
Find the side labeled x .

a)



$$\frac{\sin 105^\circ}{5} = \frac{\sin 45^\circ}{x}$$
$$x = \frac{5 \sin 45^\circ}{\sin 105^\circ} = 3.66$$

b)



$$x^2 = 145^2 + 212^2 - 2 \cdot 145 \cdot 212 \cdot \cos 52^\circ = 28118.1325$$
$$x = 167.7$$

Question 4 (15 points)

Find the amplitude, period, and phase shift of $y = 3 \cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$ and sketch the graph.

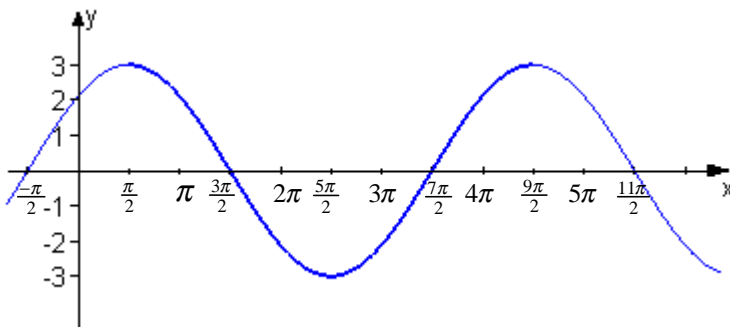
$$y = 3 \cos\left(\frac{x}{2} - \frac{\pi}{4}\right) = 3 \cos\frac{1}{2}\left(x - \frac{\pi}{2}\right)$$

Amplitude: 3

Period: 4π

Phase Shift: $\frac{\pi}{2}$

$$\left[\frac{\pi}{2}, \frac{9\pi}{2}\right]$$



Question 6 (24 points)

Prove the identity.

$$a) \frac{1 + \tan^2 \theta}{1 - \tan^2 \theta} = \frac{1}{\cos^2 \theta - \sin^2 \theta}$$

$$LS = \frac{1 + \tan^2 \theta}{1 - \tan^2 \theta} = \frac{1 + \frac{\sin^2 \theta}{\cos^2 \theta}}{1 - \frac{\sin^2 \theta}{\cos^2 \theta}} = \frac{\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}}{\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}} = \frac{1}{\cos^2 \theta} \cdot \frac{\cos^2 \theta}{\cos^2 \theta - \sin^2 \theta} = \frac{1}{\cos^2 \theta - \sin^2 \theta} = RS$$

$$b) \frac{\cos 3\theta - \cos 7\theta}{\sin 3\theta + \sin 7\theta} = \tan 2\theta$$

$$\begin{aligned} LS &= \frac{\cos 3\theta - \cos 7\theta}{\sin 3\theta + \sin 7\theta} \\ &= \frac{-2 \sin\left(\frac{3\theta+7\theta}{2}\right) \sin\left(\frac{3\theta-7\theta}{2}\right)}{2 \sin\left(\frac{3\theta+7\theta}{2}\right) \cos\left(\frac{3\theta-7\theta}{2}\right)} \\ &= \frac{-2 \sin\left(\frac{10\theta}{2}\right) \sin\left(\frac{-4\theta}{2}\right)}{2 \sin\left(\frac{10\theta}{2}\right) \cos\left(\frac{-4\theta}{2}\right)} \\ &= -\frac{\sin(-2\theta)}{\cos(-2\theta)} \\ &= -\tan(-2\theta) \\ &= \tan 2\theta = RS \end{aligned}$$