CEGEP CHAMPLAIN ST. LAWRENCE CHAMPLAIN REGIONAL COLLEGE

MATHEMATICS 201-BNK-05
Advanced Calculus
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XXIV – Applications of Fourier Series

1. Solve the wave equation $u_{tt}(x,t) = a^2 u_{xx}(x,t)$ subject to the following conditions.

   a) $a=1$, $u(0,t)=u(4,0)=0$, $u_t(x,0)=0$, $u(x,0)=\begin{cases} x & 0\le x\le 2 \\ 4-x & 2<x\le 4 \end{cases}$

   b) $a=1$, $u(0,t)=u(6,0)=0$, $u_t(x,0)=0$, $u(x,0)=\begin{cases} x & 0\le x\le 2 \\ 2 & 2<x\le 4 \\ 6-x & 4<x\le 6 \end{cases}$

   c) $a=3$, $u(0,t)=u(4,0)=0$, $u_t(x,0)=0$, $u(x,0)=\begin{cases} x & 0\le x\le 3 \\ 12-3x & 3<x\le 4 \end{cases}$

   d) $a=1$, $u(0,t)=u(4,0)=0$, $u_t(x,0)=10$, $u(x,0)=0$

   e) $a=6$, $u(0,t)=u(6,0)=0$, $u_t(x,0)=\begin{cases} x & 0\le x\le 3 \\ 6-x & 3<x\le 6 \end{cases}$, $u(x,0)=0$

2. Solve the heat equation $u_t(x,t) = k u_{xx}(x,t)$ subject to the following conditions.

   a) $k=1$, $T(0,t)=0$, $T(4,t)=0$, $T(x,0)=x$

   b) $k=2$, $T(0,t)=0$, $T(5,t)=0$, $T(x,0)=\sin\left(\frac{\pi x}{5}\right)$

   c) $k=1$, $T(0,t)=100$, $T(4,t)=100$, $T(x,0)=0$

   d) $k=3$, $T(0,t)=100$, $T(2,t)=0$, $T(x,0)=50x$

   e) $k=1$, $T_x(0,t)=0$, $T_x(4,t)=0$, $T(x,0)=x$

   f) $k=4$, $T(0,t)=0$, $T(6,t)=0$, $T(x,0)=\sin\left(\frac{\pi x}{6}\right)$

ANSWERS

1. a) $u(x,t)=\sum_{n=1}^{\infty}\frac{16(-1)^{n+1}}{\pi^2(2n-1)^2}\sin\left(\frac{(2n-1)\pi}{4}x\right)\cos\left(\frac{(2n-1)\pi}{4}t\right)$

   b) $u(x,t)=\sum_{n=1}^{\infty}\frac{12\left(\sin\frac{\pi n}{3}+\sin\frac{2\pi n}{3}\right)}{\pi^2 n^2}\sin\left(\frac{n\pi}{6}x\right)\cos\left(\frac{n\pi}{6}t\right)$

   c) $u(x,t)=\sum_{n=1}^{\infty}\frac{32\sin\frac{3\pi n}{4}}{\pi^2 n^2}\sin\left(\frac{n\pi}{4}x\right)\cos\left(\frac{n\pi}{4}t\right)$

   d) $u(x,t)=\sum_{n=1}^{\infty}\frac{160}{\pi^2(2n-1)^2}\sin\left(\frac{(2n-1)\pi}{4}x\right)\sin\left(\frac{(2n-1)\pi}{4}t\right)$

   e) $u(x,t)=\sum_{n=1}^{\infty}\frac{64(-1)^{n+1}}{\pi^3(2n-1)^3}\sin\left(\frac{(2n-1)\pi}{4}x\right)\sin\left(\frac{(2n-1)\pi}{4}t\right)$

2. a) $T(x,t)=\sum_{n=1}^{\infty}\frac{8(-1)^{n+1}}{\pi n}\sin\left(\frac{n\pi x}{4}\right)e^{-\frac{n^2\pi^2}{16}t}$

   b) $T(x,t)=\sin\left(\frac{\pi x}{5}\right)e^{-\frac{2\pi^2}{25}t}$

   c) $T(x,t)=100-\sum_{n=1}^{\infty}\frac{400}{\pi(2n-1)}\sin\left(\frac{\pi(2n-1)}{4}x\right)e^{-\frac{\pi^2(2n-1)^2}{16}t}$

   d) $T(x,t)=-50x+100-\sum_{n=1}^{\infty}\frac{400}{\pi(2n-1)}\sin\left(\frac{\pi(2n-1)}{2}x\right)e^{-\frac{3\pi^2(2n-1)^2}{4}t}$

   e) $T(x,t)=2-\sum_{n=1}^{\infty}\frac{16}{\pi^2(2n-1)^2}\cos\left(\frac{n\pi}{4}x\right)e^{-\frac{\pi^2(2n-1)^2}{16}t}$

   f) $T(x,t)=\frac{2}{\pi}-\sum_{n=2}^{\infty}\frac{4}{\pi(n^2-1)}\cos\left(\frac{\pi(2n-1)}{6}x\right)e^{-\frac{\pi^2(2n-1)^2}{9}t}$