CEGEP CHAMPLAIN ST. LAWRENCE CHAMPLAIN REGIONAL COLLEGE

MATHEMATICS 201-BNK-05
Advanced Calculus
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Winter 2011

XX - First Order Differential Equations

1. Find the order and degree for the following differential equations.

   a) $\frac{dy}{dx} + x^2 y \tan x = y^5$

   b) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = e^{x^2}$

   c) $y^{(4)} + 3\left(y''\right)^3 - y' = x^2$

   d) $\ln y'' + xy' = e^x$

   e) $\frac{y' + y}{y''' + x} = y''' + y'$

   f) $\sqrt{y'' + xy} = y'$

2. Verify that the given function is a particular solution of the given differential equation

   a) $y = 2e^x + 3e^{2x} + x^2 e^x$ $\quad y'' - 3y' + 2y = 2e^x(1 - x)$

   b) $y = \frac{x^2}{x^3 + 4}$ $\quad 2ydx - 3xy^2 dx - xdy = 0$

   c) $y = \frac{1}{2}\int_0^x f(t)\left(e^{x-t} - e^{t-x}\right)dt$ $\quad \frac{d^2y}{dx^2} - y = f(x)$

3. Sketch a direction field for the differential equation. Then use it to sketch a solution curve.

   a) $y' = x - y$

   b) $y' = xy + y^2$

4. Solve the following separable differential equations.

   a) $\frac{dy}{dx} = \left(1 + y^2\right)x^2$

   b) $\frac{dy}{dx} = y\sin(2x + 3)$

   c) $e^{-y}\sin x - y'\cos^2 x = 0$

   d) $\frac{dy}{dx} = xy^2 - x - y^2 + 1$

   e) $\frac{dy}{dx} - \frac{y^2 - y}{\cos x} = 0$

   f) $e^y \sin 2x\,dx + \cos x\left(e^{2y} - y\right)dy = 0$

   g) $\frac{dy}{dx} = \frac{4x^2}{y + \cos y}$

   h) $\frac{dy}{dx} = \frac{2x + \sec^2 x}{2y}$

   i) $\frac{dy}{dx} = y^2 - 3y - 10$

   j) $x + 2y\sqrt{x^2 + 1}\frac{dy}{dx} = 0$

5. Solve the following homogenous differential equations.

   a) $\left(x^2 + 2y^2\right)dx + 3xydy = 0$

   b) $\left(5x^3 - y^3\right)dx + 3xy^2 dy = 0$

   c) $(2x + 3y)dx + (3x + 2y)dy = 0$

   d) $\left(x^2 - xy + y^2\right)dx + 3x^2 dy = 0$

   e) $\left(x - y\sin\frac{y}{x}\right)dx + x\sin\frac{y}{x}dy = 0$

   f) $y^2 e^{\frac{y}{x}}dx + 2x^2 e^{\frac{y}{x}}dy = 0$

6. Solve the following exact or almost exact differential equations.

a) $(3y+2)dx+(3x-5)dy=0$

b) $(2xy^3+2x)dx+(3x^2y^2-6y)dy=0$

c) $(2x\sin y+y)dx+(x^2\cos y+x)dy=0$

d) $(e^x+3x^2y^2+e^y)dx+(2x^3y+xe^y)dy=0$

e) $(e^{x+3y}+\cos x\sin y+2xy)dx+(3e^{x+3y}+\sin x\cos y+x^2)dy=0$

f) $(y+2xy^2)dx-(x+3y^4)dy=0$

g) $\left(ye^{\frac{x}{y}}-2xy^3\right)dx-\left(xe^{\frac{x}{y}}+x^2y^2\right)dy=0$

h) $(3x^2y+2xy^2)dx+(2x^3+3x^2y+5y^3)dy=0$

i) $y^3dx+\sqrt{1-x^2}\,(3y^2\arcsin x+2y)dy=0$

j) $(y^2+y^3\cos^2 x)dx+(2y\sin x\cos x+3xy^2\cos^2 x-e^y\cos^2 x)dy=0$

k) $y\sec x\tan xdx+(\sec x+2\sin y\cos y)dy=0$

l) $2xydx+(1+2y^2)dy=0$

7. A differential equation $M(x,y)dx+N(x,y)dy=0$ can sometimes be made into an exact differential equation by multiplying it with an integrating factor $\mu(x,y)$. Show that you obtain and exact differential equation by using the given integration factor, then solve it.

a) $ydx-(x^2+y^2+x)dx=0$ $\qquad \mu(x,y)=\dfrac{1}{x^2+y^2}$

b) $\left(8xy^{\frac{11}{2}}+6y\right)dx+\left(10x^2y^{\frac{9}{2}}+x\right)dy=0$ $\qquad \mu(x,y)=\dfrac{x^2}{\sqrt{y}}$

8. Solve the following linear or Bernoulli differential equations.

a) $(3y-5x)dx+dy=0$

b) $\dfrac{dy}{dx}+\dfrac{3y}{x}=x^2$

c) $\dfrac{dy}{dx}+y\cot x=3e^{\cos x}$

d) $\dfrac{dy}{dx}+\dfrac{y}{x\ln x}=x^4$

e) $(xy-4)dx+(x^2-1)dy=0$

f) $\dfrac{dy}{dx}+y\tan x=\sec x$

g) $\dfrac{dy}{dx}+2y=y^3e^x$

h) $\dfrac{dy}{dx}+\dfrac{y}{x}=\dfrac{1}{x^3y^3}$

i) $\cos x\dfrac{dy}{dx}+y\sin x=3y^3\cos^2 x$

j) $(y-x^4y^4)dx+xdy=0$

k) $\dfrac{dy}{dx}=y+y^4$

l) $(x^4y-x^{14}-x^9)dx+(x^5+1)dy=0$

m) $\dfrac{dy}{dx}-y=2x\sqrt[3]{y}$

n) $x\dfrac{dy}{dx}+y=y^3x^2\ln x$

9. Solve the following differential equations.

a) $(3x+y)\,dx+(x+2y)\,dy=0 \qquad y(0)=2$

b) $\left(3y-4yx^3-x^4\right)dx+x^4dy=0 \qquad y(1)=\frac{4}{3}$

c) $2x^2y^2dx-\left(1+x^3\right)^2dy=0 \qquad y(0)=3$

d) $\left(2xye^{x^2y}+2x\right)dx+\left(x^2e^{x^2y}-3y^2\right)dy=0 \qquad y(0)=2$

e) $\left(x^4+y^4\right)dx+xy^3dy=0 \qquad y(1)=0$

f) $\left(y-y^3\sin x\right)dx+dy=0 \qquad y(0)=\frac{1}{2}$

g) $\left(2x^3+4x^2y+2xy^2+2x+2y\right)dx+(2x+2y)\,dy=0 \qquad y(0)=5$

h) $\dfrac{dy}{dx}=\dfrac{3y}{5x-xy} \qquad y(1)=1$

i) $\left(2xy^2e^y+4xy\right)dx+\left(x^2y^2e^y-2x^2\right)dy=0 \qquad y(2)=1$

j) $\dfrac{dy}{dx}+y\cot x=3e^{\cos x} \qquad y\left(\frac{\pi}{2}\right)=-1$

k) $\left(y+\sqrt{x^2-y^2}\right)dx-xdy=0 \qquad y(2)=1$

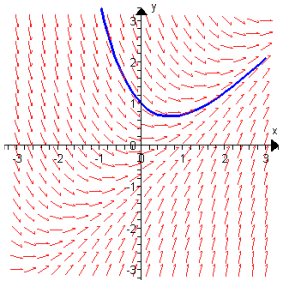
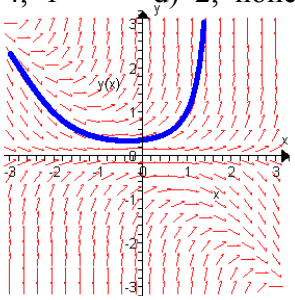
l) $(y\ln x+y-x)\,dx+ydy=0 \qquad y(1)=1$

m) $\dfrac{1}{y^4}\dfrac{dy}{dx}-\dfrac{1}{xy^3}=2+\ln x \qquad y(1)=1$

n) $\left(3x\sqrt{x^2+y^2}+2y\right)dx+\left(3y\sqrt{x^2+y^2}+2x\right)dy=0 \qquad y(3)=4$

ANSWERS

1. a) 1, 1 b) 2, 1 c) 4, 1 d) 2, none e) 3, 2 f) 2, 1

3. a)  b) 

4. a) $y=\tan\left(\frac{x^3}{3}+C\right)$ b) $y=Ce^{-\frac{1}{2}\cos(2x+3)}$ c) $y=\ln\left(\sec x+C\right)$

   d) $y=\dfrac{1+e^{x^2-2x+C}}{1-e^{x^2-2x+C}}$ e) $y=\frac{\cos x}{\cos x-C(1+\sin x)}$

   f) $ye^{-y}=2\cos x+C$ g) $y^2+2\sin y=\frac{8}{3}x^3+K$ h) $y^2=x^2+\tan x+K$

   i) $\frac{y-5}{y+2}=Ke^{7x}$ or $y=\frac{5+2Ke^{7x}}{1-Ke^{7x}}$ j) $y^2=K-\sqrt{x^2+1}$

5. a) $x^4\left(x^2+5y^2\right)^3=K$ b) $y=\sqrt[3]{Kx-\frac{5}{2}x^3}$ c) $x^2+3xy+y^2=K$

   d) $y=\frac{3x}{\ln|x|+K}-x$ e) $y=x\arccos\left(\ln|x|+K\right)$ f) $y=\frac{2Kx}{x-K}$

6. a) $3xy+2x-5y=K$ b) $x^2y^3+x^2-3y^2=K$ c) $x^2\sin y+xy=K$

   d) $e^x+x^3y^2+xe^y=K$ e) $e^{x+3y}+\sin x\sin y+x^2y=K$ f) $x+x^2y-y^4=Ky$

   g) $e^{\frac{x}{y}}-x^2y=K$ h) $x^3y^2+x^2y^3+y^5=0$ i) $y^3\arcsin x+y^2=K$

   j) $y^2\tan x+xy^3-e^y=K$ k) $y\sec x+\sin^2 y=K$ l) $ye^{x^2+y^2}=K$

7. a) $\arctan\frac{x}{y}-y=K$ b) $x^4y^5+x^3\sqrt{y}=K$

8. a) $y=Ke^{-3x}+\frac{5}{3}x-\frac{5}{9}$ b) $y=\frac{1}{6}x^3+\frac{K}{x^3}$ c) $y=-3\csc x e^{\cos x}+K\csc x$

   d) $y=\frac{5x^5\ln x-x^5+K}{25\ln x}$ e) $y\sqrt{x^2-1}=4\ln\left(x+\sqrt{x^2-1}\right)+K$ f) $y=\sin x+K\cos x$

   g) $y^2=\frac{3}{2e^x+Ke^{4x}}$ h) $y^4=\frac{2}{x^2}+\frac{K}{x^4}$ i) $y^2=\frac{\cos^2 x}{2\sin^3 x-6\sin x+K}$

   j) $y=\frac{1}{\sqrt[3]{Kx^3-3x^4}}$ k) $y=\frac{e^x}{\sqrt[3]{K-e^{3x}}}$ l) $y=\frac{1}{66}\left(x^5+1\right)\left(6x^5-5\right)+\frac{K}{\left(x^5+1\right)^{1/5}}$

   m) $y^{\frac{2}{3}}=-3-2x+Ke^{\frac{2}{3}x}$ n) $y^2=\frac{1}{Kx^2-x^2(\ln x)^2}$

9. a) $3x^2+2xy+2y^2=8$ b) $y=\frac{1}{3}x^4+x^4e^{\frac{1}{x^3}-1}$ c) $y=\frac{3+3x^3}{1-x^3}$

   d) $e^{x^2y}+x^2-y^3=-7$ e) $y=\frac{1-x^8}{2x^4}$ f) $y^2=\frac{5}{2\cos x+4\sin x+18e^{2x}}$

   g) $y=5e^{\frac{-1}{2}x^2}-x$ h) $x^3e^{y-1}=y^5$ i) $x^2e^y+2x^2=4y\left(e+2\right)$

   j) $y=-3\csc x e^{\cos x}+2\csc x$ k) $y=x\sin\left(\ln x+\frac{\pi}{6}-\ln 2\right)$ l) $y=\frac{3\left(2-2x+x^2\right)e^x-2e}{xe^x}$

   m) $y^3=\frac{2\sqrt[3]{2}x}{\sqrt[3]{-21x^4-12x^4\ln x+37}}$ n) $\left(x^2+y^2\right)^{\frac{3}{2}}+2xy=149$