CEGEP CHAMPLAIN ST. LAWRENCE CHAMPLAIN REGIONAL COLLEGE

MATHEMATICS 201-BNK-05
Advanced Calculus
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XVII – Triple Integrals

1. Evaluate the iterated integral.
   a) $\int_0^1\int_1^3\int_{-1}^2 x^3y^2z\,dzdydx$
   b) $\int_1^3\int_0^z\int_{-1}^2 xe^y\,dxdydz$
   c) $\int_0^2\int_0^{\sqrt{4-x^2}}\int_{-5+x^2+y^2}^{3-x^2-y^2} x\,dzdydx$
   d) $\int_0^4\int_0^{\frac{\pi}{2}}\int_0^{1-x} x\cos y\,dzdydx$

2. Evaluate the triple integral.
   a) $\iiint_S 2x\,dV$ where $S=\left\{(x,y,z)\,|\,0\le y\le 2, 0\le x\le\sqrt{4-y^2}, 0\le z\le y\right\}$
   b) $\iiint_S 6xy\,dV$ where S lies under the plane $z=1+x+y$ and above the region in the xy-plane bounded by the curves $y=\sqrt{x}$, $y=0$, and $x=1$.
   c) $\iiint_S x^2e^y\,dV$ where S is bounded by the parabolic cylinder $z=1-y^2$ and the planes $z=0$, $x=-1$ and $x=1$.
   d) $\iiint_S yz\,dV$ where S is the solid bounded by the cylinder $y=9-x^2$ and the planes $y+z=9$, $y=0$ and $z=0$.
   e) $\iiint_S \sqrt{x^2+y^2}\,dV$ where S is the solid determined by the conditions $x^2+y^2\le 1$ and $0\le z\le\sqrt{x^2+y^2}$.

3. Find the volume of each solid using a triple integral.
   a) The solid bounded above by $z=y$, below by $z=0$ and laterally by the cylinder $y=1-x^2$.
   b) The solid bounded by the parabolic cylinder $z-y^2=0$ and the planes $z=y-x$ and $x=0$.
   c) The solid bounded by the surface $y=x^2$ and the planes $y+z=4$ and $z=0$.
   d) The solid bounded by $y=z^2$, $z=y^2$, $x+y+z=2$ and $x=0$
   e) The solid bounded by $y=4-x^2-z^2$, $x=0$, $y=0$, $z=0$ and $x+z=2$

4. Evaluate the following improper integrals, if they converge.
   a) $\iiint_S e^{-2x-3y-4z}\,dzdydx$ where S is the first octant.
   b) $\iiint_S \frac{1}{\sqrt{x+y+z}}\,dzdydx$ where S is the unit cube in the first octant.

5. Sketch the solid whose volume is given by the integral.

a) $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{y+1} dzdydx$

b) $\int_{0}^{9}\int_{0}^{\frac{y}{3}}\int_{0}^{\sqrt{y^2-9x^2}} dzdxdy$

c) $\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{2} dydzdx$

d) $\int_{-2}^{2}\int_{0}^{4-y^2}\int_{0}^{2} dxdzdy$

6. Express the triple integral $\iiint_S f(x,y,z)\,dV$ in six different ways for the given solid *S*.

a) *S* is bounded by $x^2+z^2=4$, $y=0$ and $y=6$.

b) *S* is bounded by $z=0$, $z=y$ and $x^2=1-y$

7. Rewrite the triple integral in five different ways.

a) $\int_{0}^{3}\int_{0}^{\sqrt{9-z^2}}\int_{0}^{x} \delta dydxdz$

b) $\int_{0}^{1}\int_{0}^{y}\int_{z-y}^{y-z} \xi dxdzdy$

8. Find the centroid of *S*.

a) The tetrahedron in the first octant enclosed by the coordinate planes and the plane $x+y+z=1$.

b) The solid bounded by the surface $z=y^2$ and the planes $x=0$, $x=1$, and $z=1$.

c) The solid bounded by the cylinder $x^2+y^2=9$ and the planes $x+z=3$ and $z+1=0$.

9. Find the center of mass for the solid *S* bounded by the paraboloid $z=9-x^2-y^2$ and the plane $z=0$ if the density at any point is given by $\delta(x,y,z)=z$.

10. Find the center of mass for the solid cube in the first quadrant bounded by the coordinate planes and the planes $x=a$, $y=a$ and $z=a$ if the density at any point is given by $\delta(x,y,z)=x^2+y^2+z^2$.

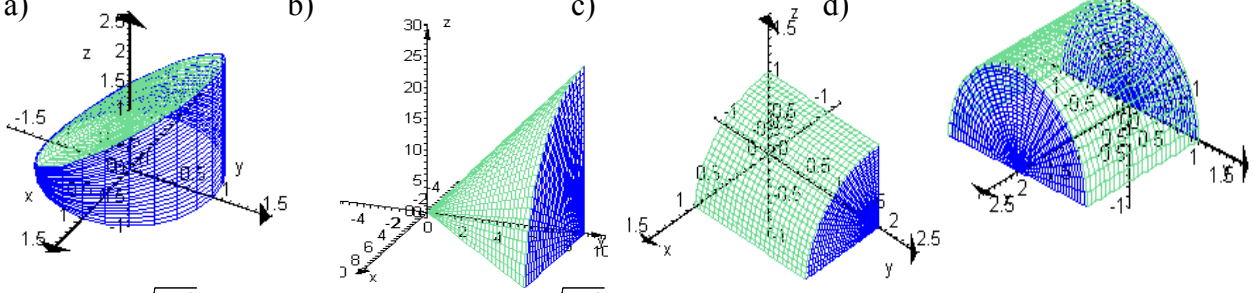
11. The average value of a function $f(x,y,z)$ over a solid region *S* is defined to be $f_{avg}=\frac{1}{V}\iiint_S f(x,y,z)\,dV$ where *V* is the volume of *S*. Find the average value of the function $f(x,y,z)=x^2z+y^2z$ over the solid enclosed by the paraboloid $z=1-x^2-y^2$ and the plane $z=0$.

12. Find the center of mass and the moments of inertia about each axis for the solid cube in the first quadrant bounded by the coordinate planes and the planes $x=a$, $y=b$ and $z=c$ if the density at any point is constant, $\delta(x,y,z)=\delta$.

13. Evaluate $\int_{-1}^{1}\int_{-x^2}^{x^2}\int_{x}^{y}\int_{\sqrt{\ln(x+y)}}^{\sqrt{z}}\int_{0}^{2\sqrt{t}} xywe^{t^2}\,dwdtdzdydx$.

ANSWERS

1. a) $\frac{13}{4}$ b) $\frac{3}{2}e^3-\frac{3}{2}e-3$ c) $\frac{128}{15}$ d) $\frac{-40}{3}$
2. a) 4 b) $\frac{65}{28}$ c) $\frac{8}{3e}$ d) $\frac{8748}{7}$ e) $\frac{\pi}{2}$
3. a) $\frac{8}{15}$ b) $\frac{1}{60}$ c) $\frac{256}{15}$ d) $\frac{11}{30}$ e) $\frac{16}{3}$
4. a) $\frac{1}{24}$ b) $\frac{8}{5}-\frac{32\sqrt{2}}{5}+\frac{24\sqrt{3}}{5}$
5. a) b) c) d)



6. a) $\int_{-2}^{2}\int_{0}^{6}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x,y,z)\,dzdydx=\int_{0}^{6}\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x,y,z)\,dzdxdy$

$=\int_{-2}^{2}\int_{0}^{6}\int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}} f(x,y,z)\,dxdydz=\int_{0}^{6}\int_{-2}^{2}\int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}} f(x,y,z)\,dxdzdy$

$=\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{6} f(x,y,z)\,dydzdx=\int_{-2}^{2}\int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}}\int_{0}^{6} f(x,y,z)\,dydxdz$

b) $\int_{-1}^{1}\int_{0}^{1-x^2}\int_{0}^{y} f(x,y,z)\,dzdydx=\int_{0}^{1}\int_{-\sqrt{1-y}}^{\sqrt{1-y}}\int_{0}^{y} f(x,y,z)\,dzdxdy$

$=\int_{0}^{1}\int_{z}^{1}\int_{-\sqrt{1-y}}^{\sqrt{1-y}} f(x,y,z)\,dxdydz=\int_{0}^{1}\int_{0}^{y}\int_{-\sqrt{1-y}}^{\sqrt{1-y}} f(x,y,z)\,dxdzdy$

$=\int_{-1}^{1}\int_{0}^{1-x^2}\int_{z}^{1-x^2} f(x,y,z)\,dydzdx=\int_{0}^{1}\int_{-\sqrt{1-z}}^{\sqrt{1-z}}\int_{z}^{1-x^2} f(x,y,z)\,dydxdz$

7. a) $\int_{0}^{3}\int_{0}^{\sqrt{9-x^2}}\int_{0}^{x}\delta\,dydzdx=\int_{0}^{3}\int_{0}^{\sqrt{9-z^2}}\int_{y}^{\sqrt{9-z^2}}\delta\,dxdydz=\int_{0}^{3}\int_{0}^{\sqrt{9-y^2}}\int_{y}^{\sqrt{9-z^2}}\delta\,dxdzdy$

$=\int_{0}^{3}\int_{y}^{3}\int_{0}^{\sqrt{9-x^2}}\delta\,dzdxdy=\int_{0}^{3}\int_{0}^{x}\int_{0}^{\sqrt{9-x^2}}\delta\,dzdydx$

b) $\int_{0}^{1}\int_{z}^{1}\int_{z-y}^{y-z}\xi\,dxdydz=\int_{0}^{1}\int_{z-1}^{0}\int_{z-x}^{1}\xi\,dydxdz+\int_{0}^{1}\int_{0}^{1-z}\int_{x+z}^{1}\xi\,dydxdz$

$=\int_{-1}^{0}\int_{0}^{x+1}\int_{z-x}^{1}\xi\,dydzdx+\int_{0}^{1}\int_{0}^{1-x}\int_{x+z}^{1}\xi\,dydzdx=\int_{-1}^{0}\int_{-x}^{1}\int_{0}^{x+y}\xi\,dzdydx+\int_{0}^{1}\int_{x}^{1}\int_{0}^{y-x}\xi\,dzdydx$

$=\int_{0}^{1}\int_{-y}^{0}\int_{0}^{x+y}\xi\,dzdxdy+\int_{0}^{1}\int_{0}^{y}\int_{0}^{y-x}\xi\,dzdxdy$

8. a) $\left(\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)$ b) $\left(\frac{1}{2},0,\frac{3}{5}\right)$ c) $\left(0,\frac{-9}{16},\frac{41}{32}\right)$
9. $\left(0,0,\frac{9}{2}\right)$ 10. $\left(\frac{7a}{12},\frac{7a}{12},\frac{7a}{12}\right)$ 11. $\frac{1}{12}$
12. $(\bar{x},\bar{y},\bar{z})=\left(\frac{a}{2},\frac{b}{2},\frac{c}{2}\right)$, $I_x=\frac{abc}{3}\left(b^2+c^2\right)\delta$, $I_y=\frac{abc}{3}\left(a^2+c^2\right)\delta$, $I_z=\frac{abc}{3}\left(a^2+b^2\right)\delta$
13. $2-\frac{1}{2}e-\frac{3}{2e}$