



MATHEMATICS 201-BNK-05

Advanced Calculus

Martin Huard

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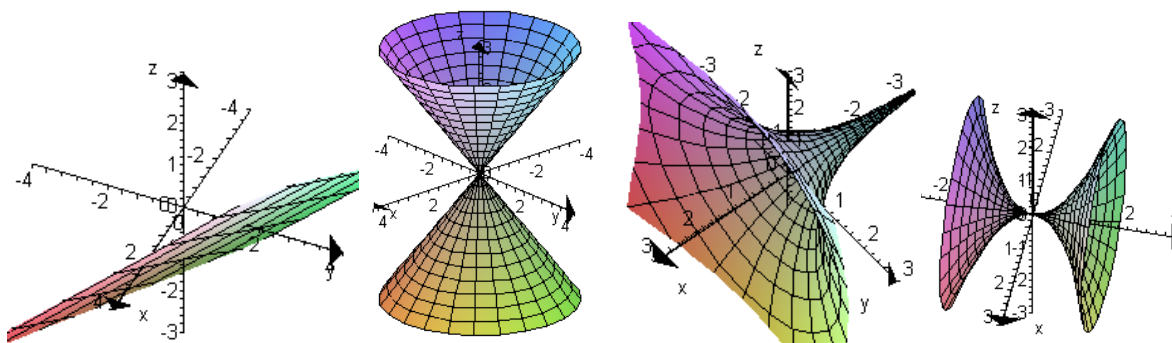
XVI - Parametric Surfaces and Surface Area

- Find a parametric representation of the following surfaces and sketch a graph.
 - $2x + 3y - 4z = 5$
 - $z^2 = x^2 + y^2$
 - surface generated by revolving the curve $y - e^x = 0$ about the x -axis.
 - surface generated by revolving the curve $y^2 = x$ about the y -axis.
- Eliminate the parameters to obtain an equation in rectangular coordinates, and describe and graph the surface.
 - $x = 3u - v$, $y = u + v$, $z = 2u + 3v$
 - $x = r \cos \theta$, $y = r \sin \theta$, $z = \sqrt{4 - r^2}$
 - $x = \sin \phi \cos \theta$, $y = \sin \phi \sin \theta$ and $z = \cos \phi$
 - $x = \cos u$, $y = v$, $z = \sin u$ $0 \leq u \leq \pi$ and $v = 0..2$
- Find the equation of the tangent plane to the given surface at the given point.
 - $x = u$, $y = v$ and $z = u^2 - v^2$ at $(3, 5, -16)$
 - $x = u^2 + v$, $y = v^2$ and $z = uv^2$ at $(1, 9, 18)$
 - $\vec{r}(u, v) = (ue^v, ve^u, uv)$ at $(0, \ln 2)$
 - $\vec{r}(\phi, \theta) = (2 \sin \phi \cos \theta, 3 \sin \phi \sin \theta, 4 \cos \phi)$ at $(\frac{\pi}{4}, \frac{\pi}{2})$
 - Surface obtained by revolving the curve $y = \arctan x$ about the x -axis, at $(1, \frac{\pi\sqrt{3}}{8}, \frac{\pi}{8})$
- Find the area of the given surface.
 - The portion of the cylinder $x^2 + z^2 = 9$ that is above the rectangle $R = \{(x, y) \mid -3 \leq x \leq 3, 0 \leq y \leq 2\}$.
 - The portion of the paraboloid $z = 4 - x^2 - y^2$ that is above the xy -plane.
 - The portion of the cone $\vec{r}(u, v) = (u \cos v, u \sin v, u)$ for which $0 \leq u \leq 2v$, $0 \leq v \leq \frac{\pi}{2}$
 - The portion of the sphere $x^2 + y^2 + z^2 = 16$ between $z = 1$ and $z = 2$
 - The portion of the cone $z^2 = 9x^2 + 9y^2$ that is above the region in the first quadrant bounded by $y = x$ and $y = x^2$.
 - The portion of $z = \ln|\sec x|$ over the region $R = \{(x, y) \mid 0 \leq x \leq \frac{\pi}{4}, 0 \leq y \leq \tan x\}$.
 - The portion of the plane $z = 24 - 6x - 3y$ in the first octant.
 - The part of the sphere $x^2 + y^2 + z^2 = 4z$ that lies inside the paraboloid $z = x^2 + y^2$.
 - The surface obtained by revolving the curve $y = \frac{e^x + e^{-x}}{2}$ about the x -axis for $0 \leq x \leq 1$.

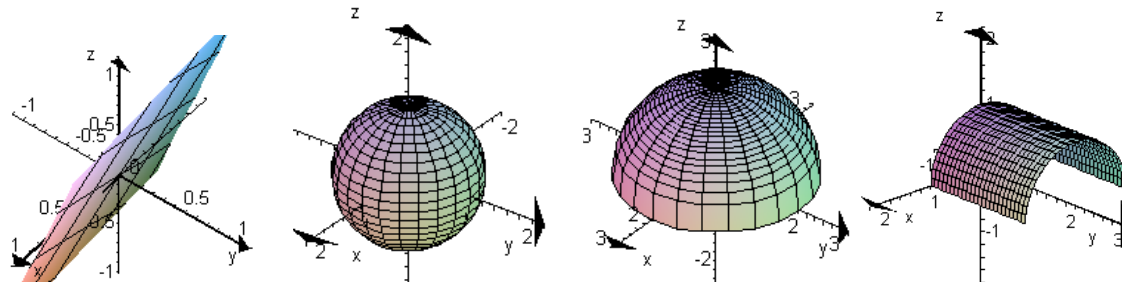
5. Find the surface area of a sphere of radius R .
6. Find the surface area of a right circular cone of radius r and height h .
7. Find the surface area of the solid of intersection of the cylinders $x^2 + z^2 = 1$ and $y^2 + z^2 = 1$.

ANSWERS

1. a)
$$\begin{cases} x = u \\ y = v \\ z = \frac{1}{2}u + \frac{3}{4}v - \frac{5}{4} \end{cases}$$
 b)
$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ z = r \end{cases}$$
 c)
$$\begin{cases} x = u \\ y = e^u \cos v \\ z = e^u \sin v \end{cases}$$
 d)
$$\begin{cases} x = u^2 \cos v \\ y = u \\ z = u^2 \sin v \end{cases}$$



2. a) Plane $z = \frac{-1}{4}x + \frac{11}{4}y$ b) Top hemisphere $z = \sqrt{4 - x^2 - y^2}$ c) Sphere $x^2 + y^2 + z^2 = 1$ d) Portion of Cylinder $x^2 + z^2 = 1$



3. a) $6x - 10y - z = -16$ b) $18x + 19y - 8z = 45$ c) $(\ln 2)x - 2z = 0$

d) $4y + 3z = 12\sqrt{2}$ e) $x - \sqrt{3}y - z = 1 - \frac{\pi}{2}$

4. a) 6π b) $\frac{17\sqrt{17}-1}{6}\pi$ c) $\frac{\sqrt{2}}{12}\pi^3$ d) 8π e) $\frac{\sqrt{10}}{6}$

f) $\sqrt{2} - 1$ g) $16\sqrt{46}$ h) 8π i) $\frac{\pi(e^4 + 4e^2 - 1)}{4e^2}$

5. $4\pi R^2$ 6. $\pi R\sqrt{R^2 + h^2}$ 7. 16