



## MATHEMATICS 201-BNK-05

Advanced Calculus

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### XIII - Double Integrals over General Regions

1. Calculate the iterated integral.

a)  $\int_{\frac{1}{4}}^1 \int_{x^2}^x \sqrt{\frac{y}{x}} dy dx$

b)  $\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{2}{\sqrt{4-y^2}} dx dy$

c)  $\int_1^2 \int_0^{y^2} e^{\frac{x}{y^2}} dx dy$

d)  $\int_0^1 \int_0^x y \sqrt{x^2 - y^2} dy dx$

e)  $\int_0^{\frac{\pi}{6}} \int_0^{\cos \theta} 3r^2 \sin \theta dr d\theta$

2. Evaluate the double integral.

a)  $\iint_R x^2 y dA$        $R = \{(x, y) \mid 0 \leq x \leq 2, 0 \leq y \leq \sqrt{4-x^2}\}$

b)  $\iint_R ye^x dA$        $R = \{(x, y) \mid 0 \leq x \leq y^2, 0 \leq y \leq 1\}$

c)  $\iint_R e^{x+y} dA$        $R = \{(x, y) \mid |x| + |y| \leq 1\}$

d)  $\iint_R x \cos y dA$        $R$  is bounded by  $y = 0$ ,  $y = x^2$  and  $x = 1$

e)  $\iint_R xy dA$        $R$  is bounded by  $y = \sqrt{x}$ ,  $y = 6 - x$  and  $y = 0$

f)  $\iint_R x^2 dA$        $R$  is the region in the first quadrant enclosed by  $xy = 1$ ,  $y = x$  and  $y = 2x$

g)  $\iint_R \sqrt{1+x^2} dA$        $R$  is the triangle with vertices at  $(0,0)$ ,  $(1,2)$  and  $(1,0)$

h)  $\iint_R x dA$        $R$  is the smaller region cut from the disk  $x^2 + y^2 = 9$  by the line  $x + y = 3$

3. Evaluate the integral by reversing the order of integration.

a)  $\int_0^1 \int_{3y}^3 e^{x^2} dx dy$

b)  $\int_0^4 \int_{\sqrt{y}}^2 e^{x^3} dx dy$

c)  $\int_0^1 \int_{-1}^{-\sqrt{x}} \sqrt[5]{y^3 + 1} dy dx$

d)  $\int_0^{\frac{1}{2}} \int_{\arcsin y}^{\frac{\pi}{6}} \csc^2(\cos x) dx dy$

e)  $\int_1^e \int_0^{\ln x} y dy dx$

4. Use a double integral to find the area of the region  $R$  in the  $xy$  plane bounded by the given curves.

a)  $y = x^2, y = 2x + \frac{5}{4}$

b)  $y = x^3, y = \sqrt[3]{x}$

c)  $x = y^2, x = y + 2$

d)  $x = y(2 - y), x = -y$

e)  $y = \frac{2}{x^2 + 1}, y = x^2$

5. Find the volume of each solid.

a) The solid bounded by the cylinder  $x^2 + y^2 = 9$  and the planes  $z = 0$  and  $z = 3 - x$ .

b) The solid in the first octant under the plane  $x + y + z = 6$  and inside the parabolic cylinder  $y = 4 - x^2$ .

c) The wedge cut from the cylinder  $4x^2 + y^2 = 9$  by the planes  $z = 0$  and  $z = y + 3$ .

d) The solid bounded by the cylinders  $x^2 + z^2 = 1$  and  $y^2 + z^2 = 1$  in the first octant.

e) The solid bounded above by the paraboloid  $z = x^2 + y^2$ , below by the plane  $z = 0$ , and laterally by  $y = x^2$  and  $y = x$ .

6. Determine if the following improper integrals converge. If so, to what value?

a)  $\int_{\frac{2}{\pi}}^{\infty} \int_0^{\frac{1}{x}} \frac{1}{x} \sin\left(\frac{1}{x}\right) dy dx$

b)  $\iint_D \frac{1}{x+y} dA$  where  $D$  is the region below the curve  $y = \frac{1}{x}$  and to the right of the line  $x = 1$ .

c)  $\iint_R \frac{1}{x^2 + y^3} dA$  where  $R$  is bounded by  $y = x$ ,  $y = -x$  and above  $y = 1$

d)  $\iint_T \frac{1}{x\sqrt{y}} dA$  where  $T$  is the triangle with vertices  $(0,0)$ ,  $(1,1)$  and  $(1,2)$

e)  $\iint_R \frac{1}{xy} dA$  where  $R$  is the region bounded by  $y = x$  and  $y = x^2$

## ANSWERS

1. a)  $\frac{83}{672}$     b) 4    c)  $\frac{7}{3}(e-1)$     d)  $\frac{1}{12}$     e)  $\frac{7}{64}$   
 2. a)  $\frac{32}{15}$     b)  $\frac{1}{2}e-1$     c)  $e-\frac{1}{e}$     d)  $\frac{1}{2}-\frac{1}{2}\cos 1$     e)  $\frac{50}{3}$     f)  $\frac{1}{8}$     g)  $\frac{4\sqrt{2}}{3}-\frac{2}{3}$     h)  $\frac{9}{2}$   
 3. a)  $\frac{1}{6}e^9 - \frac{1}{6}$     b)  $\frac{1}{3}e^8 - \frac{1}{3}$     c)  $\frac{5}{18}$     d)  $\cot \frac{\sqrt{3}}{2} - \cot 1$     e)  $\frac{1}{2}e-1$   
 4. a)  $\frac{9}{2}$     b) 1    c)  $\frac{9}{2}$     d)  $\frac{9}{2}$     e)  $\pi - \frac{2}{3}$   
 5. a)  $27\pi$     b)  $\frac{292}{15}$     c)  $\frac{27\pi}{2}$     d)  $\frac{2}{3}$     e)  $\frac{3}{35}$   
 6. a) 1    b)  $\frac{\pi}{2} - \ln 2$     c)  $\pi - 2 \ln 2$     d)  $4\sqrt{2} - 4$     e) Diverges