MATHEMATICS 201-BNK-05
Advanced Calculus
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XII - Double Integrals

1. Calculate the iterated integral.

   a) $\int_0^1 \int_1^2 (x+4xy)\,dydx$

   b) $\int_0^{\frac{\pi}{4}} \int_1^3 x^2 \sin y\,dxdy$

   c) $\int_2^4 \int_1^3 \left(\frac{y}{x}+\frac{x}{y}\right)dxdy$

   d) $\int_0^2 \int_0^3 xe^{xy}\,dxdy$

   e) $\int_1^2 \int_{-1}^1 \frac{x}{y^2-4}\,dydx$

   f) $\int_1^3 \int_1^2 \frac{1}{4x+3y}\,dydx$

2. Calculate the double integral.

   a) $\iint_R \frac{xy}{\sqrt{x^2+y^2+1}}\,dA$ $\quad R=\{(x,y)\,|\,0\le x\le 1, 0\le y\le 1\}$

   b) $\iint_R y\cos(x+y)\,dA$ $\quad R=\left\{(x,y)\,|\,\frac{\pi}{4}\le x\le \frac{\pi}{2}, \frac{-\pi}{6}\le y\le \frac{\pi}{6}\right\}$

   c) $\iint_R \frac{x}{x^2-y^2}\,dA$ $\quad R=\{(x,y)\,|\,2\le x\le 3, 0\le y\le 1\}$

3. Find the volume under the plane $2x+y-z=0$ and over the rectangle $R=\{(z,y)\,|\,1\le x\le 3, 0\le y\le 2\}$.

4. Find the volume under the hyperbolic paraboloid $z=4+x^2-y^2$ and over the square $R=\{(z,y)\,|\,-1\le x\le 1, 0\le y\le 2\}$.

5. Find the volume of the solid in the first octant bounded by the coordinate planes, the cylinder $z=9-y^2$ and the plane $x=2$.

6. Find the volume in the first octant bounded by the coordinate planes, the plane $y=4$, and the plane $\frac{1}{3}x+\frac{1}{5}z=1$.

7. Let $f(x,y)=x^2+y$ and let the rectangle $R=[0,2]\times[0,2]$ be subdivided into 16 subrectangles. Take (x_k^*, y_k^*) to be the center of the k^{th} rectangle, and approximate the double integral of f over R by the resulting Riemann sum. Also, find the exact value of the integral and compare it to the approximation.

8. Evaluate the following improper integrals, if they converge.

a) $\int_1^{\infty}\int_2^4 \frac{y}{1+x^2}\,dydx$

b) $\iint_R xye^{-x^2-y^2}\,dA$ where R is the first quadrant.

c) $\iint_R \frac{x^2+y^2}{\left(1+x^2\right)\left(1+y^2\right)}\,dA$ where R is the half strip $R=\left\{\left(x,y\right):0\le x<\infty,0\le y\le 1\right\}$

d) $\iint_R \frac{xy}{x^2+y^2}\,dA$ where R is the region given by $R=\left\{\left(x,y\right):0\le x<1,0\le y\le 1\right\}$

e) $\iint_R \frac{1}{x+y^2}\,dA$ where R is the region given by $R=\left\{\left(x,y\right):0\le x<1,0\le y\le 1\right\}$

ANSWERS

1. a) $\frac{7}{2}$ b) $\frac{26}{3}-\frac{13\sqrt{2}}{3}$ c) $6\ln 3+4\ln 2$ d) $\frac{1}{2}e^6-\frac{7}{2}$
   e) $\frac{-3}{4}\ln 3$ f) $\frac{1}{12}\ln 7-\frac{25}{12}\ln 5+\frac{2}{3}\ln 2+\frac{7}{4}\ln 3$
2. a) $\sqrt{3}+\frac{1-4\sqrt{2}}{3}$ b) $\frac{\sqrt{6}}{12}\pi-\frac{\sqrt{2}}{2}$ c) $3\ln 2-\frac{3}{2}\ln 3$
3. 20
4. 12
5. 36
6. 30
7. Approximate value: $\frac{37}{4}$ Exact value: $\frac{28}{3}$ Difference: $\frac{1}{12}$
8. a) $\frac{3\pi}{2}$ b) $\frac{1}{4}$ c) Diverges d) $\frac{1}{2}\ln 2$ e) $\ln 2+\frac{\pi}{2}$