MATHEMATICS 201-BNK-05
Advanced Calculus
Martin Huard
Winter 2011

XI - Lagrange Multipliers

1. Find the maximum and minimum values (if any) of the function subject to the given constraint(s).
   a) $f(x,y) = x^2 - y^2$; $x^2 + y^2 = 1$
   b) $f(x,y) = xy^2$; $2x^2 + y^2 = 6$
   c) $f(x,y,z) = x^2 + y^2 + 4z^2$; $2x + 3y - 4z = 34$
   d) $f(x,y,z) = x^4 + y^4 + z^4$; $x^2 + 4y^2 + 9z^2 = 36$
   e) $f(x,y,z) = xyz$; $x^2 + y^2 + z^2 = 12$
   f) $f(x,y,z) = x^2 + y^2 + z^2$; $x + y + 2z = 15$, $3x - 2y + z = 25$
   g) $f(x,y,z) = z$; $x^2 + 2y^2 = 9$, $2x + 8y + z = 5$

2. Find the point Q on the plane $\pi : 3x - 2y + z = 16$ that is closest to the point $P(11,-9,7)$.

3. Find the dimension of the rectangular box with largest volume with a surface area of 24 m^2.

4. Find the volume of the largest rectangular box with edges parallel to the axes that can be inscribed in the ellipsoid $9x^2 + 36y^2 + 4z^2 = 108$.

5. A package in the shape of a rectangular box can be mailed by Post Canada if the sum of its length and girth (the perimeter of a cross-section perpendicular to the length) is at most 36cm. Find the dimensions of the package with largest volume that can be mailed by Post Canada.

6. The plane $x + 2y + z = 6$ intersects the paraboloid $z = x^2 + 2y^2$ in an ellipse. Find the highest and lowest points on this ellipse.

7. Find the point Q on the intersection of the planes $3x - y - 5z = -14$ and $x + y + z = 10$ that is closest to the point $P(3,-1,2)$.

8. Find the point Q on the hyperplanes $x + y + z - w = 6$, $x - y = 4$ and $z + w = 2$ closest to the origin.

9. Find the maximum and minimum values for the function $f(x_1,\ldots,x_n) = x_1 + 2x_2 + \cdots + nx_n$ subject to $x_1^2 + x_2^2 + \cdots + x_n^2 = 1$.

ANSWERS

1. a) max $f(\pm 1,0)=1$, min $f(0,\pm 1)=-1$

   b) max $f(1,\pm 2)=4$, min $f(-1,\pm 2)=-4$

   c) min $f(4,6,-2)=68$ and no max

   d) max $f(\pm 6,0,0)=1296$, min $f\left(\pm\frac{3\sqrt{2}}{7},\frac{6\sqrt{2}}{7},\frac{9\sqrt{2}}{7}\right)=f\left(\pm\frac{3\sqrt{2}}{7},\frac{6\sqrt{2}}{7},-\frac{9\sqrt{2}}{7}\right)$

   $=f\left(\pm\frac{3\sqrt{2}}{7},-\frac{6\sqrt{2}}{7},\frac{9\sqrt{2}}{7}\right)=f\left(\pm\frac{3\sqrt{2}}{7},-\frac{6\sqrt{2}}{7},-\frac{9\sqrt{2}}{7}\right)=\frac{648}{49}$

   e) max $f(2,2,2)=f(2,-2,-2)=f(-2,2,-2)=f(-2,-2,2)=8$

   min $f(-2,-2,-2)=f(2,2,-2)=f(2,-2,2)=f(-2,2,2)=-8$

   f) min of $f(6,-1,5)=62$ and no max

   g) max of $f(-1,-2,23)=23$, min $f(1,2,-13)=-13$
2. $Q(2,-3,4)$
3. 2m by 2m by 2m
4. 48
5. 12 cm by 6 cm by 6 cm
6. Lowest point is $(1,1,3)$ and highest point is $(-2,-2,12)$
7. $Q(4,1,5)$
8. $Q\left(\frac{7}{2},\frac{-1}{2},\frac{5}{2},\frac{-1}{2}\right)$
9. Maximum value: $f\left(\frac{\sqrt{6}}{\sqrt{n(n+1)(2n+1)}},\frac{2\sqrt{6}}{\sqrt{n(n+1)(2n+1)}},\ldots,\frac{n\sqrt{6}}{\sqrt{n(n+1)(2n+1)}}\right)=\sqrt{\frac{n(n+1)(2n+1)}{6}}$

   Minimum value: $f\left(\frac{-\sqrt{6}}{\sqrt{n(n+1)(2n+1)}},\frac{-2\sqrt{6}}{\sqrt{n(n+1)(2n+1)}},\ldots,\frac{-n\sqrt{6}}{\sqrt{n(n+1)(2n+1)}}\right)=-\sqrt{\frac{n(n+1)(2n+1)}{6}}$