MATHEMATICS 201-BNK-05
Advanced Calculus
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VI – Limits and Continuity

1. Evaluate the following limits.

   a) $\lim\limits_{(x,y)\to(2,1)}\left(3x^2y-4x+3xy^3\right)$

   b) $\lim\limits_{(x,y)\to(0,0)}\dfrac{x^4-y^4}{x^2+y^2}$

   c) $\lim\limits_{(x,y,z)\to(3,-1,2)} e^{x-2y}\cos\frac{\pi}{2z}$

2. Evaluate the limit of $f(x,y)=\dfrac{3xy^3}{x^4+2y^4}$ as (x,y) approaches $(0,0)$ from

   a) the x-axis

   b) the y-axis

   c) the line $y=mx$ where $m\neq 0$

   d) the parabola $y=kx^2$

3. Find the limit, and, if it exists prove it using the definition, and, if not, show that the limit does not exist.

   a) $\lim\limits_{(x,y)\to(0,0)}\dfrac{5xy^2}{x^2+y^2}$

   b) $\lim\limits_{(x,y)\to(2,1)}\left(4x+3y\right)$

   c) $\lim\limits_{(x,y)\to(0,0)}\dfrac{3x^2y^4}{\left(x^2+y^4\right)^3}$

   d) $\lim\limits_{(x,y)\to(0,0)}\dfrac{2x^5+y^6+y^4}{x^4+y^4}$

   e) $\lim\limits_{(x,y)\to(0,0)}\dfrac{x^2+y^2}{x+2}=0$

   f) $\lim\limits_{(x,y)\to(0,0)}\dfrac{x^2}{x^2+y^2}$

   g) $\lim\limits_{(x,y)\to(0,0)}\dfrac{x^2y+y^3}{x^3+3y^3}$

   h) $\lim\limits_{(x,y)\to(0,0)}\dfrac{xy}{\sqrt{x^2+y^2}}$

   i) $\lim\limits_{(x,y,z)\to(3,-1,2)}\left(2x+4y-7z\right)$

   j) $\lim\limits_{(x,y,z)\to(0,0,0)}\dfrac{xy+yz+xz}{x^2+y^2+z^2}$

4. Prove that $f(x,y)=x^2+4y^2$ is continuous at $(0,0)$ using the formal definition for continuity.

5. Prove that $f(x,y,z)=5x-4y^2+z^4+2$ is continuous at $(3,-4,2)$ using the formal definition for continuity.

6. Is the function $f(x,y)=\begin{cases} yx^3+\frac{x^3}{y}+\frac{x^3}{y^2} & y\neq 0\\ 0 & y=0\end{cases}$ continuous at $y=0$? Support your answer.

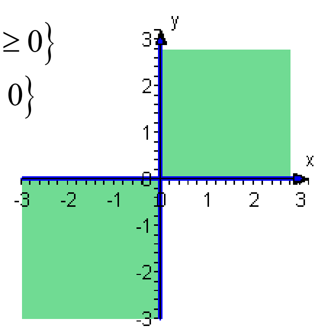
7. For each of the following functions, (i) sketch a diagram showing the domain of each function in the xy plane, (ii) specify which points of the domain are interior points, and (iii) determine those interior points of the domain – if any – at which the function is discontinuous.

   a) $f(x,y)=\sqrt{xy}$ b) $f(x,y)=\dfrac{2y+1}{x-1}$

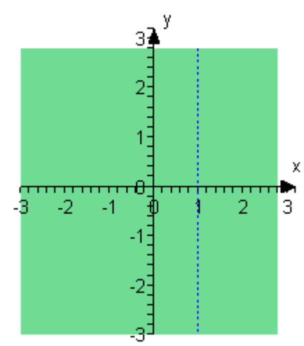
   c) $f(x,y)=\begin{cases}\dfrac{x+y}{x-y} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$ d) $f(x,y)=\begin{cases}x^2+y^2 & x^2+y^2\geq 9\\ 0 & x^2+y^2<9\end{cases}$

ANSWERS

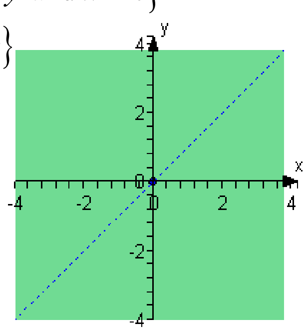
1. a) 10 b) 0 c) $\frac{\sqrt{2}}{2}e^5$
2. a) 0 b) 0 c) $\frac{3m^3}{1+2m^4}$ d) 0
3. a) 0 b) 11 c) $\nexists$ d) $\nexists$ e) 0 f) $\nexists$
   g) $\nexists$ h) 0 i) -12 j) $\nexists$
6. No since $\lim\limits_{(x,y)\to(0,0)} f(x,y)\nexists$
7. a) Dom: $\{(x,y)\mid xy\geq 0\}$
   I.P.: $\{(x,y)\mid xy>0\}$
   Cont. on all I.P



   b) Dom: $\{(x,y)\mid x\neq 1\}$
   I.P.: $\{(x,y)\mid x\neq 1\}$
   Cont. on all I.P



   c) Dom: $\{(x,y)\mid x\neq y \text{ and } x\neq 0\}$
   I.P.: $\{(x,y)\mid x\neq y\}$
   Cont on all I.P



   d) Domain: $\mathbb{R}^2$ I.P.: $\mathbb{R}^2$
   Cont. on $\{(x,y)\mid x^2+y^2\neq 9\}$, that is,
   discont. on the circle $x^2+y^2=9$