MATHEMATICS 201-BNK-05
Advanced Calculus
Martin Huard
Winter 2011

CEGEP CHAMPLAIN ST. LAWRENCE CHAMPLAIN REGIONAL COLLEGE

Semester Review

1. Use the definition of a limit to prove the following statements.
   a) $\lim_{x\to 2}\left(3x^2-4x+1\right)=5$ b) $\lim_{x\to 4}\left(\sqrt{x}+x\right)=6$ c) $\lim_{x\to 3^-}\sqrt{9-x^2}=0$
   d) $\lim_{x\to 2^+}\frac{1-x}{x-2}=-\infty$ e) $\lim_{x\to -\infty}\frac{x^2}{x^2+1}=1$ f) $\lim_{x\to\infty}\left(x^3+x\right)=\infty$
   g) $\lim_{x\to -3}\frac{x}{x^2+6x+9}=\infty$ h) $\lim_{x\to a}\left(x^2-5x\right)=a^2-5a$

2. Use the definition of a limit to show that the following statements are false.
   a) $\lim_{x\to 3}\left(x^2-x+1\right)=2$ b) $\lim_{x\to -1}\frac{x}{x+1}=\infty$
   c) $\lim_{x\to\infty}\frac{x^2}{x+1}=1$ d) $\lim_{x\to\infty}\left(x-x^2\right)=\infty$
   e) $\lim_{x\to 2^-}f\left(x\right)=5$ where $f\left(x\right)=\begin{cases}x^2 & x<2\\ 7-x & x\geq 2\end{cases}$

3. Use the formal definition for continuity to prove the continuity of *f* at the given point.
   a) $f\left(x\right)=2x^2+3x-4$ at $x=-1$ b) $f\left(x\right)=\frac{12}{x^2}$ at $x=2$

4. Show that the following functions are discontinuous at the given point.
   a) $f\left(x\right)=\begin{cases}2x+1 & \text{if } x<-2\\ x-2 & \text{if } x\geq -2\end{cases}$ at $x=-2$ b) $f\left(x\right)=\begin{cases}\frac{x-3}{x} & \text{if } x\leq 3\\ x & \text{if } x>3\end{cases}$ at $x=3$

5. Show that the functions $f\left(x\right)=\frac{1}{\sqrt{x}}$ is continuous everywhere for $x>0$.

6. Consider the curve given by $\vec{r}\left(t\right)=\left(t,\frac{1}{t},\sqrt{t}\right)$.
   a) Evaluate $\lim_{t\to 9}\vec{r}\left(t\right)$.
   b) Evaluate $\int\vec{r}\left(t\right)dt$.
   c) Find the unit tangent, normal and binormal vectors at $t=1$.
   d) Find the equation for the tangent line, the normal plane and the osculating plane, at $t=1$.

7. Consider the curve given by $\vec{r}(t)=\left(e^t-t,\sqrt{7}e^{\frac{1}{2}t},3e^{\frac{1}{2}t}\right)$.
   a) Find the arc length of the graph of $\vec{r}(t)$ from $t=0$ to $t=1$.
   b) Find the curvature at $t=0$
   c) Find the unit tangent, normal and binormal vectors at $t=0$.
   d) Find the equation for the tangent line, the normal plane and the osculating plane, at the given point.

8. Reparametrize the curve $\vec{r}(t)=\left(t^3,t^2+1\right)$ with respect to arc length measured from the point where $t=0$ in the direction of increasing *t*.

9. Find the equation of the osculating circle for the given curve at the given point.
   a) $y=\cos x$ at $(0,1)$
   b) $x^2-y^2=4$ at $(2,0)$

10. Find the velocity, acceleration and speed for the position vector $\vec{r}(t)=(\sin t,\sin 2t,\sin 3t)$ at $t=\frac{\pi}{4}$.

11. Find the position and velocity vectors for $\vec{a}(t)=\left(t,\frac{-2t}{\left(t^2+1\right)^2},\frac{1}{t}\right)$ where $\vec{v}(1)=\left(2,\frac{1}{2},1\right)$ and $\vec{r}(1)=(3,\pi,1)$.

12. Find the limit, and, if it exists prove it using the definition, and, if not, show that it does not exist.
   a) $\lim\limits_{(x,y)\to(0,0)}\dfrac{5xy^3}{x^2+3y^2}$
   b) $\lim\limits_{(x,y)\to(0,0)}\dfrac{x+y^2}{x^2+y^2}$
   c) $\lim\limits_{(x,y,z)\to(2,1,-1)}\left(x+2y-5z^2\right)$
   d) $\lim\limits_{(x,y)\to(0,0)}\dfrac{x^2y-xy^2}{x^4+y^4}$

13. Draw a contour map of the function $f(x,y)=|x|+|y|$ showing several level curves.

14. Consider the function $f(x,y)=\begin{cases}1 & (x,y)=(0,0)\\ \ln\left(x^2+y^2-1\right) & 0<x^2+y^2\le e\\ 2 & x^2+y^2>e\end{cases}$
   a) Find the domain of the function and sketch a diagram of it.
   b) Specify which points of the domain are interior points.
   c) Determine those interior points of the domain – if any – at which the function is discontinuous.

15. Find the indicated partial derivatives

a) $f(x,y)=x^3-x^2y+\sin(xy)$; f_{xy} f_{yy}

b) $f(x,y)=e^{x^2y}$; f_{yxy} f_{yxx} c) $w=z^2\sqrt{1-x^2-y^2}$; $\frac{\partial^3 w}{\partial x\partial y\partial y}$ $\frac{\partial^3 w}{\partial z^2\partial x}$

16. Show that $f(x,y)=x^3-2x^2y+y^2$ is differentiable at $(2,1)$ using the definition with ε.

17. Find $\frac{dz}{dt}$ if $z=xy^3-x^4+y^5$ where $x=\sqrt{t^2+1}$ and $y=\arctan t$.

18. Find $\frac{dw}{dt}$ if $w=x^2+y^2z-3$ where $x=\sin t$, $y=\cos t$ and $z=e^{t^2}$.

19. Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ if $z=\ln\left(x^2+xy+y^3\right)$, $x=\frac{1}{u}e^v$ and $y=u^2v^3$.

20. Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ if $z=x^3+y^2\ln x$, $x=\arcsin u^2$ and $y=u^2\arctan(4v)$.

21. Find $\frac{\partial z}{\partial t}$, $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ where $z=\sqrt{x^2+y^2}$, $x=t\sin u\cos v$, $y=\sin(u\cos t)$.

22. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial x}{\partial y}$ for $x^3y^2+xz^2-e^{yz}=x^2+z^3$

23. For $x^2+y^3+z^4=5$ and $2x-y+3z=5$, find $\frac{dx}{dz}$ and $\frac{dz}{dy}$.

24. For $x^2y^2-x+y=\sin u\cos v$ and $x^2e^{y^2}=\arctan\left(u^2v^2\right)$, find $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y}$.

25. Consider $x=\sec u\tan v$, $y=v\tan w$ and $z=\sec^2 w$. Find $\frac{\partial v}{\partial x}$, $\frac{\partial w}{\partial y}$ and $\frac{\partial w}{\partial z}$.

26. If f is a differentiable function, show that the function $w=x^2f\left(\frac{y}{x}\right)$ satisfies $x\frac{\partial w}{\partial x}+y\frac{\partial w}{\partial y}=2w$.

27. If f and g are twice differentiable functions, show that the function $w=xf(x+y)+yg(x+y)$ satisfies $\frac{\partial^2 w}{\partial x^2}-2\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial^2 w}{\partial y^2}=0$.

28. If f has continuous partial derivatives, show that the function $w=f\left(x^2-y^2,y^2-x^2\right)$ satisfies $y\frac{\partial w}{\partial x}+x\frac{\partial w}{\partial y}=0$.

29. If f is a differentiable function of two variables and $w=f\left(x,y\right)$, where $\begin{cases}x=u^2-v^2\\ y=v^2-u^2\end{cases}$ then prove that $\frac{\partial^2 w}{\partial u^2}+\frac{\partial^2 w}{\partial v^2}=\frac{u^2+v^2}{-uv}\frac{\partial^2 w}{\partial v\partial u}$.

30. Find the gradient, the gradient evaluated at the given point, and the directional derivative at the given point in the direction of the vector $\vec{u}$.
   a) $f\left(x,y\right)=\frac{xy}{x^2+y^2}$ $\quad\left(3,-4\right)$ $\quad\vec{u}=\left(2,-1\right)$
   b) $f\left(x,y,z\right)=x^2y+yz^2$ $\quad\left(2,-1,4\right)$ $\quad\vec{u}=\left(3,1,-1\right)$

31. Find the maximum rate of change of f at the given point and the direction in which it occurs.
   a) $f\left(x,y\right)=x\arctan\left(\frac{x}{y}\right)$ $\quad\left(3,-3\right)$
   b) $f\left(x,y,z\right)=\ln\left(x^2+y^2+z^2\right)-2z^2$ $\quad\left(6,-3,2\right)$

32. Find the equation of the tangent and normal lines to the curve given by $x^2y^2-x+y^5=1$ at $\left(2,-1\right)$.

33. Find the equations of the tangent plane and the normal line to each surface at the indicated point.
   a) $x^2+y^2+4=z^2$ at $\left(6,-3,7\right)$ $\qquad$ b) $y^2=x^2z^2+z+3$ at $\left(2,6,-3\right)$

34. Find the equation of the tangent line to the curve of intersection of $x^2+y^2-z^2=1$ and $5x-2y+4z=16$ at $\left(2,1,2\right)$.

35. If a triangle has two sides of length x and y, and the angle between these two sides is θ, then the area of the triangle is $A=\frac{1}{2}xy\sin\theta$. How fast is the area changing when $x=1$, $y=2$ and $\theta=\frac{\pi}{6}$, if x and y are each increasing at a rate of $\frac{1}{2}$ m/s and the angle is decreasing at a rate of $\frac{\pi}{12}$ radians per second?

36. About how accurately may the volume of a cylinder of radius r and height h be calculated from measurement of r and h that are in error of 1%?

37. Find the linearization of $f\left(x,y\right)=\arctan\left(\frac{y}{x}\right)$ at the point $\left(2,-2\right)$.

38. Find the Taylor series of the function up to and including quadratic terms.

a) $f(x,y)=e^{x^2+y^2}$ at $(2,-2)$

b) $f(x,y)=\sin(xy)$ at $\left(\frac{\pi}{6},2\right)$

39. Find the local maximum and minimum values and saddle points of the function.

a) $f(x,y)=x^3+y^3-9x^2+\frac{3}{2}y^2-18y$

b) $f(x,y)=xye^{-\frac{x^2+y^2}{2}}$

40. The origin is a critical point of $f(x,y,z)=x^2-xy+2y^2z+z^2$. Determine if $f(\vec{0})$ is a local minimum value, local maximum value, or neither.

41. Find the absolute maximum and minimum values of *f* on the set *D*.

a) $f(x,y)=48xy-32x^3-24y^2$ $D=\{(x,y)\,|\,0\le x\le 1, 0\le y\le 2\}$

b) $f(x,y)=x^2y+xy^2-x$ D is the triangle with vertices $(0,0)$, $(2,0)$ and $(0,2)$.

c) $f(x,y)=16x^2-24xy+40y^2$ $D=\{(x,y)\,|\,x^2+y^2\le 1, y\ge 0\}$

42. Find the maximum and minimum values (if any) of the function subject to the given constraint(s).

a) $f(x,y)=x^2+y$ $x^2-y^2=4$

b) $f(x,y)=x^3+y^3+z^3$; $x^2+y^2+z^2=9$

c) $f(x,y,z)=x^2+2y^4+3z^2$; $(x-2)^2+z^2=1$

d) $f(x,y,z)=xyz-x^2z$; $x^2+y^2=1,\ z=\sqrt{x^2+y^2}$

43. Find the point *Q* on the surface $z^2=xy+4$ that is closest nearest to the origin.

44. Calculate the iterated integral.

a) $\int_0^1\int_{x^2}^1 x\sqrt{x^2+y}\,dydx$

b) $\int_{-2}^2\int_0^{\sqrt{4-x^2}}\sqrt{x^2+y^2}\,dydx$

c) $\int_0^1\int_0^x \frac{x}{\sqrt{1-y^2}}\,dydx$

d) $\int_0^8\int_{\frac{y}{4}}^2 \sin(x^2)\,dxdy$

e) $\int_0^4\int_{-\sqrt{4x-x^2}}^{\sqrt{4x-x^2}} \frac{xye^{x^2+y^2}}{x^2+y^2}\,dydx$

f) $\int_0^1\int_0^{2-x}\int_2^{6-x-y^2} x\,dzdydx$

g) $\int_0^1\int_0^1\int_{x^2}^1 12xze^{zy^2}\,dydxdz$

h) $\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_0^3\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\left(y^2+z^2\right)dxdydz$

i) $\int_{-2}^2\int_0^{\sqrt{4-x^2}}\int_{-\sqrt{4-x^2-y^2}+2}^{\sqrt{4-x^2-y^2}+2}\frac{1}{\sqrt{x^2+y^2+z^2}}\,dzdydx$

j) $\int_0^1\int_0^{\sqrt{x-x^2}}\int_{x^2+y^2}^{9-x^2-y^2} xyz^2\,dzdydx$

45. Evaluate the double or triple integral.

a) $\iint_R \left(x^2y - y^2\right)dA$ where R is the region bounded by $y = 4 - x^2$ and $y = x + 2$.

b) $\iint_R \frac{1}{\sqrt{x^2+y^2}}dA$ where R is the region outside $x^2 + y^2 = 4$ and inside $x^2 + y^2 = 4x$.

c) $\iint_R \frac{x}{\sqrt{1+x^2+y^2}}dA$ where R is the region bounded by $y = \frac{1}{2}x^2$, $x = 2$ and $y = 0$.

d) $\iint_R \frac{x}{y}dydx$ where R is the region bounded below by the line $y = 1$ and above by the circle $x^2 + y^2 = 4$.

e) $\iint_R \frac{1}{\sqrt{x^2+y^2}}dA$ where R is the region to the right of $x = 1$ bounded by $x = 1$, $y = 0$ and $\sqrt{2x - x^2}$.

f) $\iiint_S xyzdV$ where S is bounded below by $z = \frac{x^2}{9} + \frac{y^2}{16}$ and above by $z = 1$.

g) $\iiint_S \left(x^2y - z\right)dV$ where S is bounded by $y + z = 1$, $z = 2y$, $z = y$, $x = 0$ and $x = 3$

h) $\iiint_S \frac{z}{\sqrt{x^2+y^2}}dV$ where S is bounded by $z = xy$, $x^2 + y^2 = 4$ and $z = 0$.

i) $\iiint_S ydV$ where S is bounded by $z = 2\sqrt{x^2+y^2}$, $x^2 + y^2 = 4$ and $z = 0$.

j) $\iiint_S xdV$ where S is bounded by $z = \sqrt{x^2+y^2}$, $z = x^2 + y^2 - 4$ and $x^2 + y^2 = 2y$.

46. Express the triple integral $\iiint_S f\left(x, y, z\right)dV$ in 6 different ways where S is the solid bounded by $z = \sqrt{y}$, $y + z = 2$, $z = 0$, $x = 0$ and $x = 2$.

47. Determine if the following integrals converge. If so, to what value?

a) $\iint_{\mathbb{R}^2} \frac{1}{x^2+y^2}dA$

b) $\iint_R \frac{x}{x^2+y^2}dA$ where R is the unit square in the first quadrant.

c) $\iiint_S \frac{1}{x^2+x^4+x^2y^2+x^2z^2}dV$ where S is the inside of the cone $z = \sqrt{x^2+y^2}$

48. Find the centroid of the given region R.

a) R is the region bounded by $y = x^3 - x$ and $y = 3x$ in the first and fourth quadrants.

b) R is the region inside the cardiod $r = 1 + \sin\theta$ and outside the circle $r = 1$.

c) R is the triangle with vertices $(1,1)$, $(2,2)$ and $(3,1)$.

49. Find the center of mass and the moments of inertia for the region in the first quadrant bounded by $y = 1 - x$ and $x^2 + y^2 = 1$ if the density is given by $\delta(x, y) = x + y$.

50. Find the volume of each solid.
  a) The solid bounded by the paraboloids $z = x^2 + 2y^2$ and $z = 12 - 2x^2 - y^2$.
  b) The solid bounded by $y = z^2$, $z = y^2$, $x + y + z = 2$ and $x = 0$.
  c) The solid bounded below by the paraboloid $z = x^2 + y^2$ and above by the plane $z = 2y$.
  d) The solid inside $x^2 + y^2 + z^2 = 12$ and outside $z^2 = 3x^2 + 3y^2$
  e) The solid bounded by $\left(x^2 + y^2 + z^2\right)^2 = x$.
  f) The solid bounded by $z = 1 - x^2$, $z = x^2 - 1$, $y + z = 1$ and $y = 0$.

51. Find the centroid of the given solid *S*.
  a) *S* is bounded by the paraboloid $z = b\left(x^2 + y^2\right)$ and the plane $z = h$ where $b > 0$ and $h > 0$.
  b) *S* is bounded by the coordinate planes $y + z = 2$ and $x = 3$.
  c) *S* is bounded above by $x^2 + y^2 + z^2 = 4z$ and below by $z = 1$.

52. Find the center of mass and the moments of inertia for the solid *S*.
  a) *S* is between the cone $x^2 + y^2 = z^2$ and the paraboloid $z = \frac{1}{4}x^2 + \frac{1}{4}y^2$ if the density is given by $\delta(x, y, z) = \sqrt{x^2 + y^2}$.
  b) *S* is the solid in the first octant bounded by the surface $x^2 + y = z$ and the planes $y = x$, $x = 1$ if the density is given by $\delta(x, y, z) = 1 + xyz$.
  c) *S* is the solid given by the equation $x^2 + y^2 + z^2 = 6z$ and if the density is given by $\delta(x, y, z) = x^2 + y^2 + z^2 + 1$.

53. Find the equation of the tangent plane to the given surface at the given point.
  a) $x = u\sin v$, $y = u^2\cos v$ and $z = u^2 - u$ at $(3, 0, 6)$
  b) $\vec{r}(u, v) = \left(u^2 + v^2, u^2 - v^2, 2uv\right)$ at $(2, 1)$

54. Find the area of the given surface.
  a) The portion of the cone $z = \sqrt{x^2 + y^2}$ that lies inside the cylinder $x^2 + y^2 = 2x$.
  b) The portion of the surface $z = 2x^2 + \sqrt{3}y$ that lies above the triangular region with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$.

55. Using a change of variables, evaluate the integral.

a) $\iint_R \left(y^2 - x^2\right) dA$ where R is the region in the first quadrant bounded by $y - x = 0$, $y - x = 1$, $xy = 1$ and $xy = 2$.

b) $\iiint_S dV$ where S is the solid bounded by the planes $x + y = 1$, $x + y = 2$, $3y - x = 0$, $3y - x = 6$, $x + y + z = 5$ and $x + y + z = 10$.

56. Use Leibnitz's rule to find the derivative of $F'(x)$. Check your result, if possible, by evaluating the integral and then differentiating.

a) $F(x) = \int_0^3 \left(x^2 y^4 + 4xy\right) dy$

b) $F(x) = \int_{x^2}^{x^3} \frac{\sin(xy)}{y} dy$

57. Given that $\int_1^c \frac{1}{u(a+bu)} du = \frac{1}{a} \ln\left|\frac{ac+bc}{a+bc}\right|$, find a formula for $\int_1^c \frac{1}{u(a+bu)^2} du$.

58. Solve the following differential equations.

a) $x\frac{dy}{dx} + 2y = 5x^4$

b) $\frac{d^2y}{dx^2} = e^y \frac{dy}{dx}$

c) $\frac{dy}{dx} = \frac{x+y}{x-y}$

d) $\left(2xy - 9x^2\right)dx + \left(2y + x^2 + 1\right)dy = 0$

e) $\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 41y = x + e^x$

f) $\left(e^{x+y} + ye^y\right)dx + \left(xe^y - 1\right)dy = 0$

g) $x\frac{dy}{dx} - 2y + x^3 y^2 = 0$

h) $\left(x \sec\frac{y}{x} + y\right)dx - xdy = 0$

i) $xydx + e^{-x^2}\left(y^2 - 1\right)dy = 0$

j) $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx} + 1\right)^2$

k) $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = e^x + \sin x$

l) $x\frac{d^2y}{dx^2} = \cot\left(\frac{dy}{dx}\right)$

m) $\left(\sin y \cos y + x\cos^2 y\right)dx + xdy = 0$

n) $2x\frac{dy}{dx} - y = x + 1$

o) $\left(y^2 - e^{-2x}\right)dx + xydy = 0$

p) $x\frac{dy}{dx} + 2y + x^3 y^2 \cos x = 0$

59. A cylindrical buoy with a diameter of $\frac{1}{2}$m and a weight of 195 kg is floating on water, its axe being vertical. A playful mermaid pulls the buoy up and down, making it oscillate. Using Archemides principle and Newton's laws, this gives rise to the differential equation.

$$\frac{d^2y}{dt^2} + y = \sin 4t \qquad \begin{cases} y(0) = -1 \\ y'(0) = 0 \end{cases}$$

Solve this differential equation.

ANSWERS

6. a) $\left(9,\frac{1}{9},3\right)$ b) $\left(\frac{1}{2}t^2,\ln|t|,\frac{3}{2}t^{\frac{3}{2}}\right)+\vec{K}$

   c) $\vec{T}(1)=\left(\frac{2}{3},\frac{-2}{3},\frac{1}{3}\right)$, $\vec{N}(1)=\left(\frac{17\sqrt{74}}{222},\frac{19\sqrt{74}}{222},\frac{2\sqrt{74}}{111}\right)$, $\vec{B}(1)=\left(\frac{-3\sqrt{74}}{74},\frac{\sqrt{74}}{74},\frac{4\sqrt{74}}{37}\right)$

   d) $l_T:\frac{x-1}{2}=\frac{1-y}{2}=z-1$, $\pi_N:2x-2y+z=1$, $\pi_O:3x-y-8z=-6$

7. a) e b) $\frac{1}{4}$ c) $\vec{T}(0)=\left(0,\frac{\sqrt{7}}{4},\frac{3}{4}\right)$, $\vec{N}(0)=(1,0,0)$, $\vec{B}(0)=\left(0,\frac{3}{4},\frac{-\sqrt{7}}{4}\right)$

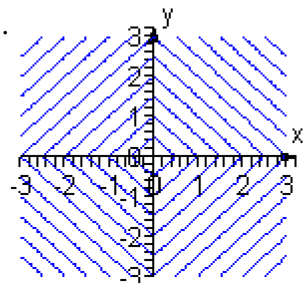
   d) $l_T:(x,y,z)=\left(1,\sqrt{7},3\right)+t\left(0,\sqrt{7},3\right)$ $\pi_N:\sqrt{7}y+3z=\sqrt{7}+3$ $\pi_O:3y-\sqrt{7}z=0$

8. $\left(\frac{1}{27}\left((27s+8)^{\frac{2}{3}}-4\right)^{\frac{3}{2}},\frac{1}{9}(27s+8)^{\frac{2}{3}}+\frac{5}{9}\right)$

9. a) $x^2+y^2=1$ b) $(x-4)^2+y^2=4$

10. $\vec{v}\left(\frac{\pi}{4}\right)=\left(\frac{\sqrt{2}}{2},0,\frac{-3\sqrt{2}}{2}\right)$ $\vec{a}\left(\frac{\pi}{4}\right)=\left(\frac{-\sqrt{2}}{2},-4,\frac{-9\sqrt{2}}{2}\right)$ $\left\|\vec{v}\left(\frac{\pi}{4}\right)\right\|=\sqrt{5}$

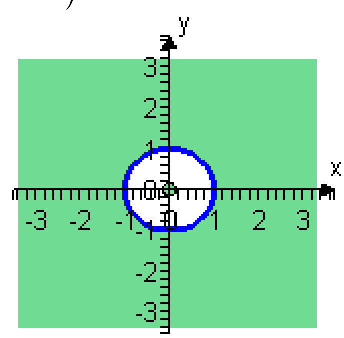
11. $\vec{v}(t)=\left(\frac{1}{2}t^2+\frac{3}{2},\frac{1}{t^2+1},\ln t+1\right)$ $\vec{r}(t)=\left(\frac{1}{6}t^3+\frac{3}{2}t+\frac{4}{3},\arctan t+\frac{3\pi}{4},t\ln t+1\right)$

12. 

13. a) $\{(x,y)\mid x^2+y^2>1\}\cup\{(0,0)\}$

    b) $\{(x,y)\mid x^2+y^2>1\}$

    c) $\{(x,y)\mid x^2+y^2>1,x^2+y^2\neq e\}$



14. a) 0 b) $\nexists$ c) -1 d) $\nexists$

15. a) $f_{xy}(x,y)=-2x-xy\sin(xy)+\cos(xy)$, $f_{yy}(x,y)=-x^2\sin(xy)$

    b) $f_{yxy}(x,y)=4x^3e^{x^2y}+2x^5ye^{x^2y}$, $f_{yxx}(x,y)=2e^{x^2y}+10x^2ye^{x^2y}+4x^4y^2e^{x^2y}$

    c) $\frac{\partial^3 w}{\partial x\partial y\partial y}=\frac{yz^2\left(y^2-2x^2-1\right)}{\left(1-x^2-y^2\right)^{\frac{5}{2}}}$ $\frac{\partial^3 w}{\partial z^2\partial x}=\frac{-2x}{\sqrt{1-x^2-y^2}}$

17. $\frac{t(\arctan t)^3}{\sqrt{t^2+1}}+\frac{3(\arctan t)^2}{\sqrt{t^2+1}}-\frac{5(\arctan t)^4}{t^2+1}-4t\left(t^2+1\right)$

18. $2\sin t\cos t-2e^{t^2}\cos t\sin t+2te^{t^2}\cos^2 t$

19. $\frac{\partial z}{\partial u}=\frac{-(2x+y)e^v+2\left(x+3y^2\right)u^3v^3}{\left(x^2+xy+y^3\right)u^2}$, $\frac{\partial z}{\partial v}=\frac{(2x+y)e^v+3\left(x+3y^2\right)u^3v^2}{\left(x^2+xy+y^3\right)u}$

20. $\frac{\partial z}{\partial u}=\frac{2\left(3x^3+y^2\right)u}{x\sqrt{1-u^4}}+4yu\ln x\arctan 4v$, $\frac{\partial z}{\partial v}=\frac{8yu^2\ln x}{1+16v^2}$

21. $\frac{\partial z}{\partial t}=\frac{x\sin u\cos v-yu\sin t\cos(u\cos t)}{\sqrt{x^2+y^2}}$, $\frac{\partial z}{\partial u}=\frac{xt\cos u\cos v+y\cos t\cos(u\cos t)}{\sqrt{x^2+y^2}}$, $\frac{\partial z}{\partial v}=\frac{-xt\sin u\sin v}{\sqrt{x^2+y^2}}$

22. $\frac{\partial z}{\partial x}=\frac{3x^2y^2+z^2-2x}{3z^2-2xz+ye^{yz}}$, $\frac{\partial x}{\partial y}=\frac{ze^{yz}-2x^3y}{3x^2y^2+z^2-2x}$

23. $\frac{dx}{dz}=\frac{-4z^3-9y^2}{2x+6y^2}$, $\frac{dz}{dy}=\frac{x+3y^2}{3x-4z^3}$

24. $\frac{\partial u}{\partial x}=\frac{2xy^2u^2v-u^2v+xe^{y^2}\sin u\sin v+xe^{y^2}u^4v^4\sin u\sin v}{u^2v\cos u\cos v+uv^2\sin u\sin v}$, $\frac{\partial u}{\partial y}=\frac{2xy^2u^2v+u^2v+x^2ye^{y^2}\sin u\sin v+x^2ye^{y^2}u^4v^4\sin u\sin v}{u^2v\cos u\cos v+uv^2\sin u\sin v}$,

$\frac{\partial v}{\partial x}=\frac{xe^{y^2}\cos u\cos v+xe^{y^2u^4v^4}\cos u\cos v-2xy^2uv^2+uv^2}{u^2v\cos u\cos v+uv^2\sin u\sin v}$, $\frac{\partial v}{\partial x}=\frac{x^2ye^{y^2}\cos u\cos v+x^2ye^{y^2u^4v^4}\cos u\cos v-2x^2yuv^2-uv^2}{u^2v\cos u\cos v+uv^2\sin u\sin v}$

25. $\frac{\partial v}{\partial x}=0$ $\frac{\partial w}{\partial y}=0$ $\frac{\partial w}{\partial z}=\frac{-1}{2}\cot w\cos^2 w$

30. a) $\nabla f(x,y)=\left(\frac{y^3-x^2y}{(x^2+y^2)^2},\frac{x^3-xy^2}{(x^2+y^2)^2}\right)$, $\nabla f(3,-4)=\left(\frac{-28}{625},\frac{-21}{625}\right)$, $D_{\vec{u}}f(3,-4)=\frac{-7\sqrt{5}}{625}$

b) $\nabla f(x,y,z)=\left(2xy,x^2+z^2,2yz\right)$, $\nabla f(2,-1,4)=(-4,20,-8)$, $D_{\vec{u}}f(2,-1,4)=\frac{16\sqrt{11}}{11}$

31. a) $\left\|\nabla f(3,-3)\right\|=\frac{1}{4}\sqrt{\pi^2+4\pi+8}$ $\vec{u}=\left(\frac{-\pi-2}{\sqrt{\pi^2+4\pi+8}},\frac{-2}{\sqrt{\pi^2+4\pi+8}}\right)$

b) $\left\|\nabla f(6,-3,2)\right\|=\frac{2\sqrt{769}}{7}$ $\vec{u}=\left(\frac{6\sqrt{269}}{5383},\frac{-3\sqrt{269}}{5383},\frac{-194\sqrt{269}}{5383}\right)$

32. $l_T: y=x-3$ $l_N: y=-x+1$

33. a) $\pi_T: 6x-3y-7z=-4$, $l_N:(x,y,z)=(6,-3,7)+t(6,-3,-7)$

b) $\pi_T: 36x-12y-23z=69$, $l_N:(x,y,z)=(2,6,-3)+t(36,-12,-23)$

34. $l_T:(x,y,z)=(2,1,2)+t(0,2,1)$ 35. $\frac{3}{8}-\frac{\sqrt{3}\pi}{24}$ m^2/s 36. 3%

37. $L(x,y)=\frac{1}{4}x+\frac{1}{4}y-\frac{\pi}{4}$

38. a) $e^{x^2+y^2}\approx e^8+4e^8(x-2)-4e^8(y+2)+9e^8(x-2)^2-16(x-2)(y+2)+9e^8(y+2)^2$

b) $\sin(xy)\approx\frac{\sqrt{3}}{2}+\left(x-\frac{\pi}{6}\right)+\frac{\pi}{12}(y-2)-\sqrt{3}\left(x-\frac{\pi}{6}\right)^2+\left(\frac{1}{2}-\frac{\sqrt{3}}{6}\pi\right)\left(x-\frac{\pi}{6}\right)(y-2)-\frac{\sqrt{3}\pi^2}{144}(y-2)^2$

39. a) rel. max of $f(0,-3)=\frac{81}{2}$, rel. min of $f(6,2)=-130$ and saddle points $(0,2)$, $(6,-3)$

b) rel. max of $f(1,1)=\frac{1}{e}$ and $f(-1,-1)=\frac{1}{e}$, rel. min of $f(1,-1)=\frac{-1}{e}$ and $f(-1,1)=\frac{-1}{e}$ and saddle point $(0,0)$.

40. No local maximum or minimum

41. a) Abs. max of $f\left(\frac{1}{2},\frac{1}{2}\right)=2$ and abs. min of $f(0,2)=-96$

b) Abs. max of $f\left(\frac{3}{4},\frac{5}{4}\right)=\frac{9}{8}$ and abs. min of $f(2,0)=-2$

c) Abs. max of $f\left(\cos\frac{5\pi}{8},\sin\frac{5\pi}{8}\right)=45$ and abs. min of $f(0,0)=0$

42. a) minimum of $f\left(\frac{\sqrt{17}}{4},\frac{-1}{2}\right)=\frac{15}{4}$

b) max of $f(3,0,0)=f(0,3,0)=f(0,0,3)=27$,

min of $f(-3,0,0)=f(0,-3,0)=f(0,0,-3)=-27$

c) min of $f(1,0,0)=1$ and max of $f(3,0,0)=9$

d) min of $f\left(\pm\frac{\sqrt{2+\sqrt{2}}}{2},\mp\frac{\sqrt{2-\sqrt{2}}}{2},1\right)=\frac{-\sqrt{2}-1}{2}$ and abs max of $f\left(\pm\frac{\sqrt{2-\sqrt{2}}}{2},\pm\frac{\sqrt{2+\sqrt{2}}}{2},1\right)=\frac{\sqrt{2}-1}{2}$

43. $(0,0,\pm 2)$

44. a) $\frac{4\sqrt{2}}{15}-\frac{2}{15}$ b) $\frac{8\pi}{3}$ c) $\frac{\pi}{8}$ d) $2-2\cos 4$ e) 0

f) $\frac{109}{60}$ g) $3e-6$ h) $\frac{21\pi}{2}$ i) $\frac{8\pi}{3}$ j) $\frac{769}{90}$

45. a) $\frac{-3429}{140}$ b) $4\sqrt{3}-\frac{4\pi}{3}$ c) $\frac{5}{4}\ln 5-1$ d) 0 e) $\sqrt{2}+\frac{1}{2}\ln 2-\ln\left(2+\sqrt{2}\right)$
f) 0 g) $\frac{1}{9}$ h) $\frac{4\pi}{5}$ i) 0 j) 0

46. $\iiint_S f(x,y,z)\,dV=\int_0^2\int_0^1\int_0^{\sqrt{y}} f(x,y,z)\,dzdydx+\int_0^2\int_1^2\int_0^{2-y} f(x,y,z)\,dzdydx$
$=\int_0^1\int_0^2\int_0^{\sqrt{y}} f(x,y,z)\,dzdxdy+\int_1^2\int_0^2\int_0^{2-y} f(x,y,z)\,dzdxdy=\int_0^1\int_0^2\int_{z^2}^{2-z} f(x,y,z)\,dydxdz$
$=\int_0^2\int_0^1\int_{z^2}^{2-z} f(x,y,z)\,dydzdx=\int_0^1\int_{z^2}^{2-z}\int_0^2 f(x,y,z)\,dxdydz$
$=\int_0^1\int_0^{\sqrt{y}}\int_0^2 f(x,y,z)\,dxdzdy+\int_1^2\int_0^{2-y}\int_0^2 f(x,y,z)\,dxdzdy$

47. a) Diverges b) $\frac{1}{2}\ln 2+\frac{\pi}{4}$ c) $\left(\sqrt{2}-1\right)\pi^2$

48. a) $\left(\frac{16}{15},\frac{208}{105}\right)$ b) $\left(0,\frac{15\pi+32}{6\pi+48}\right)$ c) $\left(2,\frac{4}{3}\right)$

49. $\left(\frac{3\pi}{16},\frac{3\pi}{16}\right)$ $I_x=I_y=\frac{2}{15}$

50. a) 24π b) $\frac{11}{30}$ c) $\frac{\pi}{2}$ d) 24π e) $\frac{\pi}{3}$ f) $\frac{8}{3}$

51. a) $\left(0,0,\frac{2h}{3}\right)$ b) $\left(\frac{3}{2},\frac{2}{3},\frac{2}{3}\right)$ c) $\left(0,0,\frac{9}{4}\right)$

52. a) $\left(0,0,\frac{23}{21}\right)$, $I_x=I_y=\frac{47104\pi}{189}$ $I_z=\frac{4096\pi}{21}$
b) $\left(\frac{4156}{5205},\frac{7481}{15615},\frac{9661}{15615}\right)$, $I_x=\frac{97943}{221760}$, $I_y=\frac{42265}{66528}$, $I_z=\frac{4277}{8640}$
c) $\left(0,0,\frac{285}{77}\right)$, $I_x=I_y=\frac{330804\pi}{35}$, $I_z=\frac{14904\pi}{7}$

53. a) $5x-z=9$ b) $5x-3y-4z=0$

54. a) $\sqrt{2}\pi$ b) $\frac{5\sqrt{5}}{6}-\frac{1}{6}$

55. a) $\frac{1}{2}$ b) $\frac{15}{2}$

56. a) $\frac{486}{5}x+18$ b) $\frac{4\sin x^4}{x}-\frac{3\sin x^3}{x}$

57. $\frac{1}{a^2}\ln\left|\frac{ac+bc}{a+bc}\right|-\frac{bc-b}{a(a+b)(a+bc)}$

58. a) $y=\frac{5}{6}x^4+\frac{K}{x^2}$ b) $\frac{e^y}{e^y+K_1}=K_2e^{K_1x}$ c) $\frac{1}{2}\ln\left(x^2+y^2\right)-\arctan\frac{y}{x}=K$
d) $x^2y-3x^3+y+y^2=K$ e) $y=K_1e^{5x}\cos 4x+K_2e^{5x}\sin 4x+\frac{1}{32}e^x+\frac{1}{41}x+\frac{10}{1681}$
f) $e^x+xy+e^{-y}=K$ g) $y=\frac{5x^2}{x^5+K}$ h) $\ln x-\sin\frac{y}{x}=K$
i) $\frac{1}{2}e^{x^2}-\frac{1}{2}y^2+\ln y=K$ j) $y=-x-\ln\left(x+K_1\right)+K_2$
k) $y=K_1e^x+K_2xe^x+K_3x^2e^x+\frac{1}{6}x^3e^x-\frac{1}{4}\cos x+\frac{1}{4}\sin x$
l) $y=x\arccos\left(K_1x\right)-\frac{1}{K_1}\sqrt{1-K_1^2x^2}+K_2$ m) $x\tan y+\frac{1}{2}x^2=K$
n) $y=x-1+K\sqrt{x}$ o) $2x^2y^2+2xe^{-2x}+e^{-2x}=K$ l) $y=\frac{1}{x^2\left(\sin x+K\right)}$

59. $y=-\cos t+\frac{4}{15}\sin t-\frac{1}{15}\sin 4t$